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The Welfare Policy Dilemma of a Negative Income Tax System: Normative Approach

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The Left-Right Wing Political Bargaining: Welfare Policy Dilemma–Subsidizing Citizens’ Low-Incomes

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Abstract: This paper examined the controversy of whether welfare policy should be just and fair to all citizens. Based on the assumptions similar to those implicit in Negative Income Tax, the resolution of the controversy did not favor increasing the social security grants and setting welfare benefits at overly high levels. In order to investigate this issue, two key actors/politicians and an implicit partaker took part in a case study, whereby they participated in negotiations for the well-being of citizens. While both politicians campaigned for public goods and services, their emphases were interpersonally incomparable. The first actor, representing left-wing politicians, struggled for citizens’ legal and moral rights to primary needs and for the delivery of basic goods. On the other hand, the second actor represented right-wing politicians and advocated for citizens’ needs for the delivery of non-primary but vital public goods and services. Finally, the implicit partaker, who embodied the taxpayers, preferred personal consumption to moral and social understanding. The aim of the taxpayers was minimizing their tax obligations through voting maneuvering, which could bring the negotiations at risk of premature collapse.

Keywords: bargaining; welfare policy; public goods; taxation; voting

JEL Classifications: C78, H21

1. Introduction

As the welfare policy of the state presupposes the existence of both a functioning market economy and a democratic political system, its hallmark is that the distribution of public goods and services is governmental responsibility and obligation. The term public in this context refers solely to income redistribution. In particular, an obligation to ensure that those on low-incomes are awarded appropriate levels of social benefits and subsidies ensures a more egalitarian allocation of wealth than can be provided by the free market. In this scenario, politicians face a dilemma of whether such allocation is just and fair to all citizens. The solution depends on many factors, including the characteristics and views of the main benefactors of wealth distribution.

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Political competition, related to income redistribution, often fosters debate regarding what the state "should" or "should not" deliver. Wider and more substantial welfare benefits and subsidies could be problematic, as they might encourage certain behaviors, such as low savings or productivity when economic security is guaranteed. Similarly, they may lead to high wage demands as an incentive to remain in employment, in particular when unemployment benefits are substantial and are compensated by high tax. Such high benefits typically undermine social and geographical mobility. In addition, high taxes become incentive for entering a black labor market that avoids paying high taxes, or moonlighting (i.e., holding multiple jobs). Finally, few would opt for working and studying because it simply does not pay them to do so. In sum, excessive benefits might result in the human capital not developing quickly and well enough, c.f., "...implicit support to those waiting on benefits looking for the 'right type of job' or a job that pays well enough," Oakley & Saunders [29], 2011, p. 20.

The primary goal of this case study is to express the view that arguments often advocating in favor of higher benefits and subsidies are false. Beyond the negative perception of higher benefits, it is also reasonable to believe that income distribution is, perhaps, the only target for control and an exclusive source of information for assessing the amount of benefits available. Due to the absence of panel data, based on credible distribution, and experimental support and justification for our view, we are approaching this issue from a more theoretical perspective. However, despite this key weakness, the solution of the welfare policy dilemma, based on numerical simulations, yields benefits sufficiently close to be considered a realistic match (see Table 1) to "what amounts to a moving poverty line at 50% of median income," c.f., Fuchs [16] point; Bowman [5], 1973, p. 55. In support of this approach, it is worth noting that Rawls [33], in 1971/2005, on p. 98, pronounced the Fuchs point as an alternative to the measurement of poverty with no reference to social position. The motive of the current study is thus to point—while acknowledging that a few examples clearly cannot make a trend—that we presented a theoretical confirmation for the claim recognizing the poverty line, defined as 50% of the median income, as a realistic political consensus.

In our scheme, citizens earning low-incomes (below a certain level) receive subsidies, whereas those with higher incomes (above the aforementioned level) do not. In this regard, it should be noted that, in 1962, Milton Friedman [14], 2002, pp. 190-195, proposed a similar scheme of income redistribution, combined with flat tax, called the negative income tax—the NIT. According to the rules and norms of the NIT, low-income citizens receive a subsidy proportional to the difference between their earnings and the predetermined NIT poverty line. Most importantly, the total—the sum of the key income and the NIT subsidy—is not subject to taxation. When levying taxes in compliance with of the tax rules and norms in force for all, inclusive of low-income citizens, the arithmetic of our approach would have the same result. Although the total income of low-income citizens is now taxable, they would still be eligible for subsidies in the NIT spirit, similar to the widely adopted LI—low-income—subsidies. The known drawback of such an approach, and the LI subsidies in particular, stems from the known issue of social abuse among low-income citizens. In order to mitigate its effects, we introduce the so-called hazard of working incentives, referred to as the $h$-effect.
We consider a masquerade of life or scenario of realistic utopia—a theoretical model of visionary politicians. In this scenario, two actors/politicians, akin to two political coalitions, are playing a bargaining game, each trying to implement his/her wealth distribution policy. Left-wing politicians tend to see the disproportion in private consumption, unjust redistribution of income, profit motive, and private property as the main sources of socioeconomic evil. Right-wing politicians, on the other hand, tend to focus on regulating business and financial risks, while limiting the government's use of its powers in combating corruption, criminal violence and commercial fraud. While left-wing politicians prefer an equitable sharing of the available stock of goods and services here and now, both sides are aware of the taxpayers' sacrifices—in terms of direct contribution of a part of their income to the funding of welfare benefits and public goods. However, applying the rules and norms of welfare arithmetic pertaining to the reliance on the elevated LI subsidies would increase the quantity of the subsidies to be delivered. Consequently, all citizens will have to meet a greater tax burden. This is not ideal, given that the tax burden and private consumption always lie at the heart of citizens' economic and political wants. These wants lead to conflicts of interests among taxpayers that, as voters, hold power in electing political parties. As a result, they are instrumental in the competition between two key representatives—left- and right-wing politicians—and their views on tax policies.

We hope that these brief remarks have clarified some goals of the state, allowing us to conclude that welfare policy in a representative democracy always faces conflicting interests of politicians. The aim of this case study is also to shed light on how a political consensus is reached and whether it reflects a criterion of tax policy that ensures the least burden for all citizens. To address this issue, as already stated, we focus our analysis on two visionary politicians. For the purpose of the following discussion, we granted these politicians a political mandate to initiate proposals that ensure that subsidies are allocated to citizens that are in need. In addition, we will assume that, expenses shall be constrained on subsidies in balancing the books accounting for finance public goods and services. This ensures that negotiations are under the control of the taxpayers, forcing the politicians to act within the imposed budget-constraint in order to pledge safe funding for their proposals. Hereby, while trying to reduce the after-tax income inequality, the politicians in the roles of left- and right-wing actors are committed to ensuring that the tax revenue is redistributed fairly. Here, we refer to the tax revenue as the "wealth-pie," which can be cut by the politicians into two slices \((x, y)\), \(x + y = 1\), only.

All the way through discussions, we adopt quantitative measurement, whereby we utilize a scale quantum as an average per capita of income \(\sigma\) over distribution \(P(\sigma, \xi)\), \(0 < \sigma < \infty\). The average establishes the ratio scale. Setting the poverty line \(\xi\) parameterizes variables and targets functions at the specified scale. Doing so, we suggest the after-tax residue \(u(\xi, x)\) of an income \(\sigma = \xi\) of the poverty line \(\xi\), \(\xi \in [\xi_1, \xi_2]\), to represent the left-wing politicians' targets/emphases. Similarly, the right-wing politicians' targets \(g(\xi, y)\) are the non-basic goods per capita. The third actor—the implicit partaker embodying the taxpayers with income \(\sigma\) behind the scene—votes for minimizing the
tax obligation $\tau(\sigma,x)$ and against excessive public spending. This is typical public finance dilemma, which is represented by the alternating-offers bargaining game on how to allocate by slicing $(x,y)$ the wealth-pie/tax revenue with premature risk $q$, $0 < q << 1$, of breakdown. This game is thoroughly described in the literature—the classical variant of Osborn and Rubinstein [30], 1990, p. 31. We do not go beyond the classical scheme, so there is no need for a special description of the game. It should only be noted that the opposite side would at once accept the original proposal of each player.

When negotiating on finance issues, under the guise of a "wealth-pie workshop" the politicians will allegedly try to slice the wealth-pie in a rational and efficient manner. In doing so, the politicians will meet the tax $\tau(\sigma,x)$ increase—so does the wealth-pie, increasing the poverty line $\xi$. Decrease of taxes yields the reverse effect. However, while taxes vary, the slicing will depend upon the characteristics and emphases of the bargainers performing it. Indeed, the right-left wing politicians' wants/targets $u(\xi,x)$ and $g(\xi,y)$ have been marked as being-different. We illustrate this situation by elevated single-peaked frontier of targets $u(\xi,x)$, the $\frac{7}{8}$-slice in Figure 1, which corresponds to the lower, but progressively increasing, concave frontier of targets $g(\xi,y)$, the $\frac{5}{8}$-slice in Figure 2, as well as for another allotment of the pie, into slices $(x = \frac{5}{8}, y = \frac{5}{8})$. We believe, that, while $(x = \frac{5}{8}, y = \frac{5}{8})$ highlights the left-wing emphases, the allotment $(\frac{1}{8}, \frac{1}{8})$ highlights those of the right-wing politicians. This premise is crucial for understanding our primary goal in resolving the welfare policy dilemma.

![Figure 1. Left-wing politicians emphases](image1)

![Figure 2. Right-wing politicians emphases](image2)

The single peakedness $u(\xi,x)$ of the left-wing politicians' targets/emphases emerges when the terms of contract commit politicians to slices $(x,y)$ that will balance the books, provided that the expenses related to the LI subsidies are covered by flat taxes. In support of the aforementioned belief, the politicians' emphases and views in general, shaped in this way, emerge within a two-man economy endowed by citizens' income abilities marginalized at the level of LI poverty line. According to Black [4], 1948, p. 27, single peakedness plays a major role in collective decision-making when the decision is arrived at by vote. Here, an income equal to the LI poverty line that embraces subsidies is referred to as the poverty line $\xi$. In the same vein, the citizen at the $\xi$ level is referred to as framed citizen.
The emphases of two actors, as shown in Figure 1 and Figure 2, are called non-conforming. As a result, slicing the pie no longer represents any traditional bargaining procedure as, instead of slicing, the procedure can be resettled. We can proceed at distinct levels of one parameter—inside the poverty line interval $[\xi_1, \xi_2]$, reflecting the scope of negotiations. In fact, in 2007, Cardona and Ponsatti [8], p. 628, also noted that "the bargaining problem is not radically different from negotiations to split a private surplus," when all the parties in the bargaining process have the same, conforming emphases. This argument applies even when the emphases of the first player are principally non-conforming, i.e., single-peaked, rather than concave. In our case study of non-conforming emphases, the scope of negotiations on the "contract curve" allows for omitting the axiom of "Pareto efficiency," known since Roth [37], 1977, as "well defined bargaining problem." Thus, the well-defined problem of slicing $(x, y)$ the pie, as a substitute, can be solved, instead, inside the poverty lines' interval $[\xi_1, \xi_2]$.

It is now high time to clear up some of the assumptions/limitations in the analysis of a hypothetical behavior of citizens appointed for three distinct roles in the negotiations—as those of left-right wing politicians and voters-taxpayers. Throughout the entire study, we stressed the incomparability between the targets of the left-wing politicians struggling to ensure adequate access to basic goods, and the right-wing politicians advocating for non-primary but vital goods and services. In the analysis, we implicitly assumed that politicians did not have adequate knowledge about peoples' needs in more primitive environment at their disposal and could only work with the monetary targets specification. For example, we assumed that politicians did not have any information on how the income of households was assembled and used to buy private health insurance or services of nursing housing, etc. Thus, politicians could not know that the provision of equivalently valued public services was not a perfect substitute. Therefore, we were not interested in how the optimal behavior of the taxpayers contributed to the optimum distribution $P(\sigma, \xi)$ of income $\sigma$; all these and similar questions were left unanswered. In short, we did not merit the debate on what was right or wrong in the economic or political environment involving left-right wing politicians and voters-taxpayers. Despite significance of individual characteristics of voters, we simplified voters' behavior and paid attention only to binary voters voting either for left- or right-wing politicians. Enrichment of voters' characteristics would spoil beyond doubt the delicate balance between the motives of our case study and the theoretical framework, which already is technically sophisticated.

Finally, we confirm that, rather than collecting data through voting, which would indicate proposal approval rates, we designed a debating and voting platform for the welfare policy on poverty. This approach does not require analysis of politicians' power composition, neither voting system nor a scheme by which voters-citizens express their argument as taxpayers. Instead, we adopted the view of Roberts, [36], 1977, p. 329, who noted, "The point is not whether choices in the public domain are made through a voting mechanism but whether choice procedures mirror some voting mechanism." Thus, we adhere to all voting guidelines where tax is implicit in each proposal—for example, "the unanimity rule, coupled with the proposal that each public good be financed by a separate tax constituted Wicksell's new principle of taxation"—Mueller, [24], 2003, p. 67.

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1 i.e., incomparable, impossible to match through a monotone transformation, Narens and Luce [26], 1983.

2 Johan Gustaf Knut Wicksell (20/10/1851 – 3/5/1926) was a leading Swedish economist of the Stockholm school.
Roadmap. Because of the narrative complexity, the reader, apparently, will be faced with the inability to trace the material of the paper in one chain, moving from previous to the following section. Therefore, to ensure smooth moving, the Section 2 will help us to identify the problems most relevant, in particular, the Pre-equity condition of breakdown of the negotiations. We recommend, first to learn the material of Section 2.1, and only then, if it is desirable, to move along Section 4. When mastered Section 2.2, it will be useful to succeed with Section 5, and finally, when mastered Section 2.4 complete the motion along Section 6. With respect to Section 2.3, our scheme here involves the pre-equity of breakdown of the negotiations and in contrast does not require further clarification.

Before delving deeper into the analysis based on the targets of politicians, in Section 3, we specify the category of targets functions \( u(\xi, x) \), \( g(\xi, y) \) and \( \tau(\sigma, x) \) required for the model validity, which we used in our wealth-pie workshop. In Section 4, we disclose in details fiscally safe welfare policy in amalgamation with imposed budget-constraint for financing LI subsidies. Referred to as volatility-constraint, the amalgamation dynamically restricts the \( h \)-effect—an inverse working incentives phenomenon of low-income citizens. In Section 5, taxpayers' ambivalence and multifaceted welfare policy perception are discussed from the alternating-offers game perspective. Our case study here associates the monetary targets \( u(\xi, x) \) and \( g(\xi, y) \) of the left- and right-wing politicians with policy on poverty. Therefore, in principle, given arbitrary income distribution \( P(\sigma, \xi) \), it would be possible, within the scope of negotiations \( [\xi_1, \xi_2] \), to obtain an exact analytical solution to the game. The discussion of the results is presented in Section 7, while concluding remarks are given in Section 8.

2. Relevant trends and issues

As the state has the duty to help the less fortunate, our case study approaches income redistribution in a two-fold manner. First, it addresses the provision of basic necessities or goods, such as shelter, clean and fresh water, and nutrition, etc., before focusing on non-basic goods, including national defense, public safety and order, and roads and highway systems. Welfare policy economists, c.f., Reder [34], 1947, believe that just and fair income redistribution is achieved through the efficient allocation of society's resources. Therefore, when designing welfare policy, based on income source distribution \( P(\sigma, \xi) \), while trying to redistribute income \( \sigma \) fairly, we must identify an efficient approach to the allocation \( (x, y) \), \( x + y = 1 \), of basic and non-basic goods. Fundamentally, the efficient allocation—in our workshop, the wealth-pie slicing \( (x^*, y^*) \)—aims at just and fair delivery of all aforementioned goods, preserved traditionally as public goods. In our case study, we refer to public goods as non-basic but vital goods, in contrast to, treating basic goods as fundamental. Incidentally, during the delivery of basic and non-basic goods to their end destinations, we treat both as public goods.

Suppose that, within the poverty interval \( [\xi_1, \xi_2] \), the left-wing politicians have the necessary political power—when an offer is made, irrespective of its originator—to control the poverty line \( \xi \) independently. Given the single-peaked emphasis of the, in contrast to those of their right-wing counterparts, the power the left-wing politicians enjoy is supposed to be strong enough to reach the peak of their emphases. In making these suppositions, we agree with Rawls' [33] statement, 1971/2005, p. 304, about the precepts of justice: "The sum of transfers and benefits [...] from essential public goods should be arranged so as to enhance the emphases of the least favored consistent with the required saving and the maintenance of equal liberties."
With these remarks in mind, any procedure of negotiating on slices, accompanied by poverty lines, can be perceived as two sides of the same bargain’s portfolio. Therefore, it is irrelevant whether the players are bargaining on slices of the pie or trying to agree on the position of poverty lines. This highlights the main advantage of the parametric procedure—it brings about a number of different patterns of interpretations of outcomes in the game, such as linking an outcome to the lowest tax rate, the lowest amount of taxable income, etc., all of which are the taxpayers’ most desirable sacrifices. In consideration of alternative ways—which describe outcomes of collective bargaining in the form of voting, or partaking in any voting scheme in the form of bargaining—the scope of negotiations brings the voting and bargaining schemes under one roof, both of which can be enriched by adopting this approach. Our initiative could also serve to unify the theoretical structure of economic analysis of productivity problem. Indeed, reg. Leibenstein [21], Altman [1], 2006, p. 149, wrote:

Leibenstein (1979, 493) argued that there are two components to the productivity problem: one relates to the determination of the size of the pie, while the second relates to the division of the pie. Looked upon independently, all agents can jointly gain by increasing the pie size..."the situation need not be a zero-sum game. Tactics that determine pie division can affect the size of the pie. It is this latter possibility that is especially significant.

In the extant literature, the welfare policy issues are usually addressed separately. However, in our view, a much deeper analysis is obtained addressing them together than addressing them separately. In particular, our wealth-pie workshop concept, adopting jointly four issues—(a) public finance, (b) alternating-offers game, (c) negotiations’ collapse analysis, and (d) political design—leads to a more narrative bargaining procedure. The procedure is forward-looking in the sense that it aims to find a classical solution to the game, whereby both actors accept at once the proposals made by the other side.

To explain the root cause of the results and find such bilaterally acceptable solutions to the game, we will try to visit all of the classrooms in our workshop, in order to bring the economic and political content to the surface in a rigorous analytical form. Our goal is to lay the foundation for a more constructive welfare policy comprehending the meaning of following four narratives:

- **Fiscal policy**: During the delivery to its final destinations, the wealth-pie (the wealth or tax revenue), provided that the books accounting for the LI subsidies finance have been balanced *a priori*, must remain balanced throughout and in spite of volatility in the economy;
- **Negotiations**: The left-right political bargaining on how to slice the wealth-pie complies with the rules and norms of the alternating-offers game;
- **Pre-equity of breakdown**: Breakdown, or threat, point directly affects the bargaining solution. Pre-equity guarantees equal conditions for players before the bargaining game starts;
- **Political design**: Bringing a motion to a vote is necessary to address consumers’ opposition to high taxes and excessive public spending. Whether it is viewed as positive or negative, or whether it ought to be acknowledged or not, rejected or accepted, this motion must be politically designed in advance.
In our wealth-pie workshop, these four narratives can be understood as obligations/constrains to be met by welfare policy rules and norms, c.f. "Rational man" deliberation, Rubinstein [39], 1998, p. 7. This interpretation will enable us to affirm the view under which the narratives are embedded into the welfare policy of the state. In evaluating the welfare policy from this perspective allows us to demonstrate that the analysis can be subject to and performed by computer simulations, see Appendix A2. Our initiative could also serve to unify the theoretical structure of economic analysis of public spending, evaluate the conduct of left- and right-wing politicians, or conduct systematic inquiry into impacts of governmental decisions and actions.

2.1. Fiscal policy

The efficiency of public resources implies that a consensus between left- and right-wing politicians might be reached. Despite various views to the contrary, we posit that the bargaining aimed at finding a just and fair allocation of basic vs. non-basic goods is an acceptable path to the bargaining dynamics. In particular, beyond some peak position, we believe that the greater demands on basic goods would lead to the excessive decline in the quality of welfare services, as well as cause deterioration in access to all basic goods for all citizens. Based on this premise, we review relevant publications on economic and political behavior, which deal with the sociological and political wants of welfare using public finance. In our view, this is likely the best starting point for visiting our wealth-pie workshop.

Public finance focuses on the revenue side of tax policy. In particular, it pertains to the budget formation, c.f., Formby and Medema, [15], 1995, aiming to provide a guaranteed level of welfare to citizens endowed by poor productivity. While the welfare policy is a separate matter, it is worth considering on the grounds of legal and moral rights of citizens. Empirical evidence consistent with legal obligations can be found in the literature on social policy. For example, as noted by Saunders [40], 1997, "...poverty line. The line was initially set (in 1966) equal to the level of the minimum wage plus family benefits for one-earner couple with two children." Similarly, a hypothesis consistent with moral obligations can be found in the literature of economic politics, Eichenberger & Oberholzer-Gee [11], 1996, Feld & Frey [13], 2002.

Musgrave, [25], 1959, examined two basic approaches to taxation, namely the "benefit approach" and "ability-to-pay," which put taxation into efficiency and equity context, respectively. We intend to augment the existing standard of welfare policy by benefit approach, whereby we allocate a guaranteed amount of wealth for minimum taxes. Furthermore, to ensure that taxes remain fairly levied, we posit that the tax system that maximizes wealth is based on injecting optimal equity according to the ability-to-pay principle of "proportional sacrifice."
Taxation is a principal funding source of social costs and benefits. Thus, the first postulation in our welfare policy workshop (see above) discloses an obvious paradigm in social policy. According to the ability-to-pay principle commonly adopted in public finance, in order to make the distortion of tax policies stable, the known terms of warranty must rely on exogenous taxes enforced on the productivity of citizens. The concept, proposed in 1996 by Berliant and Page Jr., [3], is a variant of the classic public finance and similar approaches, applicable when an agent characterized by a specific level of productivity does not shift his/her labor supply after all adjustments to the tax formula have been implemented. In other words, optimal taxation enforces optimal labor supply.

Yet another "treatment of policies," closely related to societal instability, entails equity of pre- and post-tax positions of taxpayers. Such a view that demarcates between citizens attracts the attention of economists and tax policy makers. In the view of Kesselman and Garfinkel, [20], 1978, credit tax-scheme analysis opposes the income-tested program in the rich-and-the-poor, two-man economy. Poverty measurements have also been addressed in the works of Sen, [41], 1976, Atkinson, [2], 1987, Ebert, [10], 2002, and Hunter et al., [18], 2002. Horizontal inequalities seem to occupy a place in Stewart's paper, [42], 2000, where the author reviewed the connection between income redistribution and economic growth. In a later study, Peñalosa and Wen, [32], 2004, investigated income redistribution, treating it as a form of social insurance. According to Tarp et al., [43], 2002, p. 8, "The poverty line acts as a threshold with households falling below the poverty line considered poor and those above poverty line considered nonpoor."

While we continue to rely on stabilization policy, we refer to welfare policy as idempotent, in order to highlight the policy particular type of the dynamics stability. For clarity, a choice operation (or decision) applied multiple times is deemed idempotent if, beyond the initial application, it yields the same result, c.f., Malishevski [23], 1998, p. 422. Thus, based on this definition, an idempotent decision scheme guarantees for politicians the ability to keep the pledges made during the election campaign as, once the decision is taken, no subsequent destabilization is possible. In the attempt to assess and control the circulation of wealth through social and public organizations, we argue that, unless dynamic stabilization is not a required condition when justifying political decisions through public spending, it will be difficult to explain how the benefits and subsidies can reach all members of the society.

2.2. Negotiations

Bargaining is the key element of economics and the core of politics. However, “The interface between economics and politics is still in a primitive state in our theories but its development is essential if we are to implement policies consistent with intentions,” as pointed out by North, [28], 2005, p. 29. Feldstein, [12], 2008, p. 132, also noted: “Unfortunately, there is no reason to be pleased about the analysis in policy discussions of the efficiency effects...of the welfare consequences of proposed tax changes.” Similarly, Richter, [35], 2005, p. 387, in a review on “Handbook of New Institutional Economics” stressed, “…that the sociological analysis...and large institutional structures in economic life is still at an early stage...game theory, and computer simulation could help to further develop the new institutional approach...game theory might be a defendable heuristic device of NIE.”
It is realistic to imagine a scenario in which our actors play the “bargaining drama” of economic and political issues in our wealth-pie workshop. While visiting the workshop, the circulation of wealth is supposed to be dynamically stable. Under these conditions, the left- and right wing politicians would be trying to implement their vision of the state welfare institutions: “These flimsy structures, however, are used by individuals to allocate resource flows to participants according to rules that have been devised in tough constitutional and collective-choice bargaining situations over time,” Ostrom, [31], 2005, p. 823. Bargaining has been a theme of a wide range of publications, including Alvin E. Roth, [38], 1985. It can be risky to bargain for both actors because the voters may defect to the other side if their terms are not met. This binary position of the voters fits particularly well into the Osborn and Rubinstein’s bargaining game with exogenous risk $q$, $0 < q < 1$, of breakdown, [30], 1990, p. 71.

We will first clarify the asymmetric dynamics of political power of both left-wing politicians (hereafter referred to as LWP, the 1st actor) and right-wing politicians (RWP, the 2nd actor). Indeed, numerous factors—such as economic growth, decline or stagnation, demographic shift or pit, political change or change in scarcity of resources, skills and education of the labor force, etc.—might often create fiscal imbalance in a desirable welfare policy due to the transfers of LI benefits and subsidies. Consequently, the size of the wealth-pie might be too small (i.e., not worth the effort), or too large (introducing mutual traps) to achieve a stabilized public spending mechanism. Thus, the actors may decide not to slice the pie at all. To address this situation, as previously underlined, we assume that LWP possess all the relevant skills of a welfare policy implied. If the policy does not meet their goal, LWP are sufficiently skilled to promote their own understanding of income redistribution to deliver the wealth "properly." For example, LWP can enforce decisions by inflaming public anger, retaliate for breaches by recruiting volunteers and movements, solicit private contributions, etc. In other words, as Kalai, [19], 1977, p. 131, put it, they would rely on an "enthusiastic supporter." On the other hand, as RWP are assumed to be lacking welfare skills and competence, they cannot fully control LWP’s actions and intentions when their political interests in the final agreement are incomparable. In these circumstances, RWP lack such abilities and knowledge and might resort to agreeing with, or at least not resist, LWP’s privileges to make arrangements upon the size of the pie. Hence, from the RWP’s critical point of view, whether acting politically in common interest or not, it might be prudent to admit the lack of knowledge and skill. This clarifies the asymmetric dynamics of political power between the LWP and RWP.

We recall the main points of bargaining scenario. The axiomatic bargaining theory finds Nash [26] solution by maximizing the product of actors' targets above the disagreement point $d = (d_1, d_2)$:

$$\text{arg max}_{0 \leq x + y \leq 1} f(x, y, \alpha) = (u(x) - d_1)^\alpha \cdot (g(y) - d_2)^{1-\alpha},$$

which is known as the asymmetric variant of the problem from Kalai [19], 1977.
Although game theory purists might find the solution above clear, the questions asked by many often include: What are $x$, $y$, $\alpha$, $u(x)$, and $g(y)$? What does the point $\langle d_1, d_2 \rangle$ mean, and how is the arg max formula used? The simple answer can be given as follows:

- $x$ is the 1st actor's slice of the pie, with $\alpha$ as the 1st actor's asymmetric power indicator, $0 \leq x \leq 1$, $0 \leq \alpha \leq 1$;
- $u(x)$ denotes the 1st actor's targets, for instance $u(x) \equiv x$, of the 1st actor's pie slice;
- $y$ is the 2nd actor's slice of the pie, where $1 - \alpha$ is the 2nd actor's asymmetric power indicator, $0 \leq y \leq 1$;
- $g(y)$ denotes the 2nd actor's targets, for instance $g(y) \equiv \sqrt{y}$, of the 2nd actor's slice of the pie.

Based on the widely accepted nomenclature, we refer to $s = \langle u(x), g(y) \rangle$ as to the utility pair. The disagreement point $d = \langle d_1, d_2 \rangle$ thus represents the payoffs the two actors collect if they cannot agree on how to slice the pie. In the same vein, $d = \langle d_1, d_2 \rangle = \langle 0, 0 \rangle$ represents the disagreement or breakdown point, whereby the players collect nothing. Let us assume that the pie has been sliced so that both actors receive equal share, $(x, y) = (\xi_1, \xi_2)$. In such a division, the half is more valuable for the 2nd actor, as his/her utility $g(\xi_2) = \sqrt{\xi_2} = 0.707$ is greater than the 1st actor's utility $u(\xi_1) = 0.5$.

The well-defined variant of asymmetric problem in our case study emerges due to the fact that we handled analytical solution for utilities $\langle u(x), g(y) \rangle$ within the scope of negotiations $[\xi_1, \xi_2]$ comprising the endpoints of the interval $[\xi_1, \xi_2]$. We succeeded to drop the allotment variables $(x, y)$ from all targets functions. If necessary, utilities/targets will be used later in the form $\langle u(\xi), g(\xi) \rangle$. Akin to the above, political bargaining in the wealth-pie workshop can now be expressed similarly by maximizing the product of politicians' targets above the threat point $d = \langle d_1 = u(\xi_1), d_2 = g(\xi_2) \rangle$:

$$\arg\max_{\xi \in [\xi_1, \xi_2]} f(\xi, \alpha) = (u(\xi) - d_1)^\alpha \cdot (g(\xi) - d_2)^{1-\alpha}.$$ 

The question of whether the poverty line $\xi \in [\xi_1, \xi_2]$ is valuable can thus be answered by solving a well-defined problem dropping the allotment variables $(x, y)$. Unlike the traditional threat point $d$, the public/vital goods amount $d_2$ in the wealth-pie game—the $d_2$ component of the point $d$—might be negative. This will apply in our case study of a breakdown of negotiations, whereby borrowing or creating money is needed to balance the books for accounting welfare expenses—a situation of "genuine negative taxes". In the theory of public finance, the use of genuine negative taxes is not prohibited.

2.3. Pre-equity of breakdown

Beyond the asymmetric dynamics, the game inherits a premature disagreement or breakdown point, similar to that discussed by Osborn and Rubinstein [30], 1990, p. 72:

"We can interpret a breakdown as the result of the intervention of a third party, who exploits the mutual gains. A breakdown can be interpreted also as the event that a threat made by one of the parties to halt the negotiations is actually realized. This possibility is especially relevant when a bargainer is a team (e.g. government), the leaders of which may find themselves unavoidably trapped by their own threats."
In our wealth-pie game, the asymmetric solution incorporates a breakdown policy into the left-right wing political power indicators \((\alpha, 1 - \alpha)\). In order to be addressed properly, the indicators cannot be given exogenously. To overcome this obstacle, we supply the game with a policy of endogenously extracted breakdown \(d = \{d_1, d_2\}\), on a condition referred to as the pre-equity of breakdown.

Traditionally, in the alternating-offers game, the breakdown corresponds to two standard pairs of payoffs \(\{1,0\}, \{0,1\}\), or in the words of Osborn and Rubinstein [30], 1990, p. 73, “to the worst outcome.” In the left-right political bargaining, due to the implicit pressure from the voters, as both politicians aim to find—at least from their perspective—a just and fair solution, there will always be a temptation for binary voters to defect to the other side. This puts the negotiations at risk \(0 < q << 1\) of a premature collapse. Even under these assumptions, the quality and the size of the wealth-pie in the event of collapse should be equal for both politicians. This premise holds as, in the worst circumstances, the entire pie will be decided upon by one of the politicians and will reflect his/her emphases. Therefore, when the premature collapse occurs, it is important to arrange the terms of contract in such a way that neither politician can exploit or misuse the collapsed environment to his/her advantage. To meet this condition, when normalizing the standard breakdown under the description valid for the alternating-offers game \(\Gamma(q)\), [30], we are working toward an endogenous form for equity, in accordance with politicians’ non-conforming targets.

As stated, the standard breakdown in the alternating-offers game corresponds to two pairs \(\{1,0\}, \{0,1\}\) of utilities. In this form, the breakdown is generally found using ex-ante linear transformation—namely, the exogenous normalization of utilities. When the likelihood of a collapse increases, the standard breakdown exposes politicians’ equity. Thus, once the most unfavorable result of negotiations occurs, the resulting collapse includes additional parameters—the tax \(\tau\) and the taxable income \(W\) per capita. In order to equalize—endogenously normalize—the breakdown, the politicians involved in negotiations can make a priori arrangements, or sign binding agreements upon these parameters \(\tau\) and \(W\). However, without availability or warranty of such a pre-equity, an endogenous normalization is unrealistic. Therefore, in the view of the taxpayers’ electoral maneuvering (discussed in the next subsection), even if the pre-equity normalization is not always achievable, it is more constructive to determine the breakdown according to some rational definition of wealth. In the absence of a universal definition, we use the term wealth narrowly defined as "prosperity or a commodity" delivered through tax channels, and distributed by the State.

Before proceeding with a detailed assessment of the aforementioned definition, we will refer to taxable income per capita as wealth amount \(W\). Next, according to the conditions characterizing the collapsed environment, at the start of the negotiations, the draft of a contract covers taxes \(\tau\) and—in line with our nomenclature—the wealth amount \(W\). The product \(\tau(\xi) \cdot W(\xi)\) identifies the size \(z\) of the pie within an interval \([\xi_1, \xi_2]\)—within the scope of negotiations, thus establishing the boundary for the two politicians. The lower limit, \(\xi_1\), denotes the initial proposal, which is the most attractive for RWP, while being the most unattractive for LWP. Similarly, the upper limit, proposal \(\xi_2\), yields exactly the opposite outcome, as \(u_t = u(\xi_2)\) is the most favorable for RWP, and \(g_t = g(\xi_1)\) is the most unfavorable for LWP. In the same but inverse order \(u_2 = u(\xi_2)\) can be paired with \(g_2 = g(\xi_2)\). Having set these limits, we can proceed with examining how the breakdown \(\{u_1, g_1\}, \{u_2, g_2\}\) might be conditionally, albeit endogenously, encoded into the wealth-pie game.
Indeed, we now contribute to implementing our wealth definition of how the breakdown can be established endogenously. To do so, we consider a situation driving the welfare policy in the context of cost-benefit equity. When the collapse of negotiations is looming closer, the differences in the amounts of wealth and taxes for funding low-cost welfare policy $\xi_1$ against an expensive policy $\xi_2$, $\xi_1 < \xi_2$—that is, funding targets $\langle u_1, g_1 \rangle$ for $\xi_1$ against $\langle u_2, g_2 \rangle$ for $\xi_2$, $u_1 < u_2$, $g_1 > g_2$—can amplify misunderstandings and contribute to traps. At the endpoints of the scope $[\xi_1, \xi_2]$, the wealth-pie sizes $z(\xi_1)$ and $z(\xi_2)$ at poverty lines $\xi_1$ and $\xi_2$ can require the delivery of wealth amounts $W(\xi_1)$ and $W(\xi_2)$, albeit at different prices, represented as taxes $\tau(\xi_1)$ and $\tau(\xi_2)$, c.f., Buchanan, [6], 1967, 4.7.1. Hence, prior to the start of the game, following cost-benefit equity, in the most adverse circumstances, the targets $s_1 = \langle u_1, g_1 \rangle$ and $s_2 = \langle u_2, g_2 \rangle$ should preserve equal prices $\tau$ for the delivery of equal amounts $W$ of wealth. Such a market-driven interpretation of commodities delivery to the end destinations relies heavily on the size of the pie, which equals $\tau \cdot W$. It should be noted that this interpretation is only relevant to the situation of flat (proportional) taxes.

To explicate the interpretation of reasoning in previous lines, it is worth examining the "well defined bargaining problem," depicted as the contract curve in Figure 5. Based on the discussion so far, our goal is to fix an interval $[\xi_1, \xi_2]$ solving two non-linear equations, $W(\xi_1) = W(\xi_2)$ and $\tau(\xi_1) = \tau(\xi_2)$, by attempting to find a cross-point $(W^*, \tau^*)$ where the curve crosses its own contour on the plain with $W, \tau$ as $XY$-axis coordinates (i.e., the roots $\xi_1^*$ and $\xi_2^*$). Although the calculus of the point $(W^*, \tau^*)$ is realistic, it does not confirm the possibility of normalization in general. However, this does not invalidate our discussion, as we do not claim that the equity condition can be reached in all circumstances. It should still be pointed out that, in a number of examples where the validity of the condition was detected, we found a breakdown endogenously encoded into the wealth-pie game, i.e., indicating normalization in the form of $\{u_1^* = u(\xi_1^*), g_1^* = g(\xi_1^*), \langle u_2^* = u(\xi_2^*), g_2^* = g(\xi_2^* )\}$.

In accord with the above, as the aim is to bring the politicians, if possible, into just and equal positions prior to negotiations, equalizing wealth amounts $W$ and taxes $\tau$ in the collapsed environments $\xi_1$ and $\xi_2$ might be a rational starting point. Endogenously encoded into the wealth-pie game, we label the equity condition $[\tau(\xi_1) = \tau(\xi_2), W(\xi_1) = W(\xi_2)]$ as a pre-equity of breakdown. If valid, this condition equalizes fiscally realistic and just demands for public spending prior to negotiations—in particular, the size of the wealth-pie $z(\xi_1) = z(\xi_2)$.

2.4. Political power design

Only the voting results can reveal the true incentives of the taxpayers, who will give the democracy its final judgment. The voting process is the only avenue for the taxpayers to assume the roles of current or upcoming politicians to whom the opportunity will be granted in accord with people’s wants to redesign the rules and norms of welfare arithmetic. Voters’ inequalities, life plans, background, social
class and experience, native endowments, political capital, etc., determine the bulletin collected at the voting table. Consequently, voters’ reasonable disagreements or interpretations of reality affect the individual choices and thus the voting results, thereby impacting politicians’ pre-election maneuvering. Voting results are not fully predictable due to the deviations in voters’ views and opinions on how the income redistribution ought to be organized. The problem stems from the fact that welfare policy proposals that benefit all citizens sometimes require higher taxes. However, taxpayers—as a subgroup of all voters—would be primarily guided by selfish attitudes toward lower taxes, which would implicitly affect the politicians’ bargaining positions. Such an attitude deserves, perhaps, a critical examination. Given these points, our question is: why should either left- or right-wing politicians care about lower taxes?

At this point, it is timely to introduce political maneuvering, with an implicit risk $q$, $0 < q < 1$, of negotiations suffering a premature collapse. Indeed, Figure 5 depicts the contract curve of efficient public policies/proposals $\xi$ upon poverty lines in the bargaining game $\Gamma(q)$. Rational politicians’ idempotent proposals $\xi$, forming the curve, have been projected onto the two-dimensional space of taxable income (the wealth amount) $W(\xi)$ and the tax rate $\tau(\xi)$. Although the pair $\langle u(\xi), g(\xi) \rangle$ is embedded in each point, it is not visible on the graph. These invisible pairs, in the upper part of the graph symbolize wealth-pie allotments $(x, y)$ of lower basic yet higher public goods, as the right-wing politicians’ targets $g$. Similarly, the pairs in the lower part symbolize reverse allotments—the lower public vs. higher basic goods, as the left-wing politicians’ targets $u$, c.f., Figure 1-2. Thus, once all views are represented, the politicians’ targets $\langle u(\xi), g(\xi) \rangle$ for pledged tax hikes $\tau(\xi)$ are more favorable for some coalitions of voters compared to others. As voters’ preferences for the balance between basic and public goods vary, the approach to determining efficient poverty lines resulting from eventual agreement between politicians is two-fold. Indeed, unless the tax hikes are excessively high, the "upper coalitions" of voters will always try to outmaneuver the "lower coalitions." The politicians are aware of this dynamic when taxes are high. As they feel trapped in negotiations, the result might be that binary voters defect to other side, putting the negotiations at risk $q > 0$ of premature collapse. In contrast, when tax is sufficiently low, the range of eventual political outmaneuvers will shrink or even vanish. Therefore, the lowest tax is likely the most desirable outcome for all citizens.

We state that the politicians on the opposite sides of the bargaining table might disagree with respect to the terms of outcomes, in which situation they would delay the decision while consolidating a draft of a consensus document. Thus, this premature agreement might not necessarily yield the best outcome for the taxpayers. In accord with the bargaining procedure, we settle in Section 6 a bargaining problem, enabling the taxpayers to accept the lowest tax $^3$ by politically designing in advance the power indicator $\alpha$ of left-wing politicians where the magnitude of the risk of collapse does not exceed $q > 0$.

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$^3$ N.B. The lowest tax is just one of the various fiscal policies that affect the design of power indicator $\alpha$. 

3. Settings

Having discussed the reasons for slicing the public goods into basic and other wants, it is deemed reasonable to follow the same pattern in reality, whereby the left-wing politicians negotiate with the right-wing politicians' favorable portions, i.e., the slices $x$, $0 \leq x \leq 1$, of the pie. Following the traditional rules of how to slice the wealth-pie in the alternating-offers game, when the pie is desirable for both sides, the politicians (bargainers)—changing roles—commit to slices $(x,y)$, $x + y = 1$. According to the slices $(x,y)$, the valid rules and norms of welfare arithmetic, which guarantee a desirable level of LI subsidies, require establishing an LI parameter $\xi$. In defining the parameter $\xi$ in this manner, it becomes contingent on financing LI subsidies. This can be achieved by assuming that higher values of the poverty line $\xi$ require an increased marginal tax rate $\tau(\sigma,x)$. Therefore, while increasing the wealth through tax channels, we assume a positive acceleration $0 > \tau'_x(\sigma,x)$ for all citizens, inclusive of $\sigma = \xi$, which indicates the framed income $\xi$ of a citizen denoted as $\sigma$.

In the above, we assume that the amount $r \cdot (\xi - \sigma)$, $0 < r \leq 1$, compensates for the unfair subsistence of the less fortunate citizen $\sigma < \xi$ and serves as a monetary compensation designated for purchasing an eligible "poverty basket" of food, clothing and shelter, fuel and lights, etc. According to Rawls, [33], 1971/2005, p. 92, "primary goods are things which it is supposed a rational man wants whatever he wants." Thus, we define the left-wing politicians' targets using the function $u(\xi,x)$, where $u(\xi,x)$—known as the poverty line residue—is the after-tax income of the framed citizen, which is equal to the poverty line level $\xi$. In this scenario, all expenses, except the LI subsidies, incorporate the right-wing politicians' targets $g(\xi,x)$, referred to as "public goods." With the proviso that politicians commit to the slice fixing $x$, we can further assume that function $u$ is a single ∩-peaked upon $\xi$ increase. Moreover, while targets $g$ of right-wing politicians decrease with $x$, they increase with elevating $\xi$. Both targets $(u,g)$ (see Figure 1 and 2) are considered analytic functions $u(\xi,x)$, $g(\xi,x)$, described as follows. Given the previously mentioned interval $[\xi_{1} \leq \xi \leq \xi_{2}]$, subsequently referred to as the scope of negotiations, $u$ is assumed to be a single ∩-peaked, $u''_\xi < 0$ upon $\xi$ increase, $u'_\xi(\xi_{1},x) > 0$, $u'_\xi(\xi_{2},x) < 0$. In addition, upon an increase in $x$, the utilities $u$ become convex, $u''_\xi > 0$, whereas following the increase in $\xi$, utilities $g$ should be concave with $g''_\xi > 0$. With increasing $x$, utilities $g$ always decrease; in other words, in both circumstances, either $g''_\xi > 0$ is convex, or $g''_\xi < 0$ is concave.

4. Fiscally safe welfare policies

The delivery of basic goods, which counteracts negative contingency, if it occurs, is the left-wing politicians' main responsibility. Herewith, the left-wing politicians' intervention is of the greatest political importance. It is universal in the sense that it pertains to all citizens, regardless of one's situation
before or after the contingency. Under this premise, basic goods that are available to citizens are of sufficiently high quality and poverty is not allowed, as stressed by Greve [17], 2008, p. 58. This course provides a relatively high level of welfare spending and tax, creating misbalance in the books accounting for public finances, i.e., introducing volatility conditions into the tax revenue refund. Hence, secured largely independently of market forces, the high level of basic goods might have a conflict-driven effect on the welfare policy, which should not be borne by taxpayers alone as, as already noted, the state has a duty to help the disadvantaged.

Assumed that the conflict-driven welfare policy guided our political actors in trying to sign an agreement, the left-wing politicians aimed to secure an efficient size of the wealth-pie. Thus, they both prescribed the size of the pie and proposed the slicing method, which the right-wing politicians accepted or rejected. If they rejected the previous proposal, the $RWP$ suggested their own version of slicing, while only having the authority to recommend a size that the $LWP$ might not be obligated to accept. We also assumed that, upon delivery to its end destinations, the wealth-pie remained fiscally safe. Under the rules of the alternating-offers procedure (see later), the game will continue until a consensus is reached. This, however, ensures that left-wing politicians are committed to the slice, albeit without being committed to the size.

Let us now envisage a contrasting scenario, whereby the public spending increases. Thus, both actors know that, upon delivery, the size of the wealth pie might change. This, in turn, leads to a misbalance between the LI subsidies and tax revenue, which can put the pie size in doubt or make it even fuzzier. The difficulty related to politicians’ pledges might thus force both sides to retreat. In such volatile conditions, the wealth-pie is no longer fiscally safe and might affect the targets of both politicians. Therefore, a fiscally safe plan for delivering and slicing the tax revenue pie is needed. Otherwise, unless welfare policy fails to enforce fiscal safety, the rules and norms of the LI are not living up to their claims. Thus, having a criterion for determining whether a welfare policy is fiscally safe is necessary.

It is helpful to focus first on welfare policy without any warranty of fiscal safety. It could, for example, be determined by the poverty line $\xi$, identifying the recipients of wealth redistribution. Thus, when $\xi$ is low, the variable $\sigma$, $0 < \sigma \leq \xi$, allocates the income of the needy or the benefit claimants and vice versa. In this scenario, the benefit claimant $\sigma < \xi$ claims and receives a benefit or subsidy proportional to $\xi - \sigma$ (i.e., $r \cdot (\xi - \sigma)$, as previously discussed), while all other citizens (both the wealthy and those with framed income, denoted as $\sigma > \xi$ and $\sigma = \xi$, respectively) receive a zero subsidy.

Next, we study a specific scheme highlighting the readiness of the society to fund welfare and public spending. For this analysis, we assume that the average cost $B$ of the LI subsidies and the average taxable income $W$ both depend on the LI parameter $\xi$, $B \equiv B(\xi)$, $W \equiv W(\xi)$ (this is realistic, as shown in Appendix A1). As previously narrowly defined, $W(\xi)$ can refer to the wealth amount. Based on our perception of incomes distribution samples $P(\sigma, \xi)$ the product $r \cdot W(\xi)$ estimates the average tax revenue. Let the average cost of public goods be $g(\xi)$, whereas the size $z(\xi)$ of the pie
equals $\tau \cdot W(\xi)$, $z(\xi) = \tau \cdot W(\xi)$. We assume that welfare and public spending reached its recipients—namely, the total spending equals $\tau \cdot W(\xi) = B(\xi) + g(\xi)$. This suggests that the basic and non-basic goods have been delivered to their final destinations. In other words, the wealth collected through tax channels is spent.

Now, let us assume that politicians in the game preferred to commit to the slice fixing $x$, and might agree to hold the balance $B(\xi) = x \cdot \tau \cdot W(\xi)$ of the books accounting for financing the LI subsidies $B$. That is, the left-wing politicians must be ready to finance the LI subsidies (i.e., to refund $B(\xi)$ via tax revenue $\tau \cdot W(\xi)$). Hereby, the politicians pledged to keep the balance $B(\xi) = x \cdot \tau \cdot W(\xi)$ of the subsidies between credits $B(\xi)$ and debts $x \cdot \tau \cdot W(\xi)$ as a portion $x$ of the tax revenue $\tau \cdot W(\xi)$. The balance also specifies the welfare policy $\xi$—an implementation of the poverty line $\xi$, welfare reform, pact, program, etc. While the balance is initially valid, it might not be in the future, putting the adjustment in $\xi$ on the agenda either once or repeatedly. Thus, the policy $\xi$ might represent a problem of fiscal imbalance. However, almost all citizens—even if for different reasons—will prefer the opposite in the long run: a fiscally safe policy $\xi$. For this reason, we now shift the focus on examining a constraint that corresponds to fiscal safety of welfare policy $\xi$, identifying—what we called above as idempotent—the safe delivery of the pie to its end destinations.

4.1. Idempotent rules and norms

The delivery of basic and public (non-basic) goods does not necessarily safeguard the funding of the expenses. Usually, as the expenses neither match nor prevent taxation hikes, the size of the wealth-pie could vary too rapidly. This leads, as previously discussed, to numerous adjustments of welfare policy rules and norms. To mitigate this issue, we have to look at the sequence $\ldots, \xi', \xi'' \ldots$ of multiple adjustments of the LI parameter $\xi$. This highlights the fact that, on delivery, the adjustments of the pie are undesirable. Consequently, it is better to keep the size of the pie unchanged, i.e., fiscally safe. In other words, when replacing the old policy $\xi'$ with $\xi''$, the two must coincide. Similar schemes, known as idempotent, stem from bounded rationality mechanisms. This suggests that, even if welfare policy rules and norms are subject to multiple adjustments, this should not change the machinery of how the LI subsidies are paid out. In particular, when implemented twice, the rules must produce the same outcome. Therefore, to guarantee the fiscal safety of the poverty line, such an understanding requires that the lines must coincide amid a sequence of pairs $(\xi', \xi'')$ at some matching policy $(\xi' = \xi'')$.

The need to balance the books accounting for the delivery of LI subsidies $B(\xi) = x \cdot \tau \cdot W(\xi)$, in spite the wealth-pie volatility, can also be seen as immunity for financing the welfare policy. In particular, the immunity holds down, or at least realistically limits the $h$-effect of income redistribution. Given the immune, i.e., fiscally idempotent, composition $[B(\xi), W(\xi)]$, the idempotent scheme is equivalent to implementing the policy $\xi$ only once. For this reason, we assume that the rules and norms of the LI subsidies have been socially planned and redesigned accordingly.
In this idempotent mode that outlines the fiscal safety of public spending, the rules and norms reflect idempotent policy $\xi$ that bring negotiations to a conclusion. We therefore conclude that the expenses $x \cdot \tau \cdot W(\xi)$ designated for welfare spending must be in balance not only for funding LI subsidies $B(\xi)$, when the particular policy $\xi$ takes effect, but also the policy $\xi$ must enforce the fiscal safety in the full spectrum of current and future events.

Clearly enough, the balance $B(\xi) = x \cdot \tau \cdot W(\xi)$ is a static relationship leading to functional dependency $\tau \equiv \frac{B(\xi)}{x \cdot W(\xi)}$ binding the variables $\xi$ and $x$. Hereby, the tax rate $\tau$ becomes a function of $\xi$ and $x$, expressed as $\tau \equiv \tau(\xi, x)$. According to Malcomsons' [22] model, 1986, p. 266, the personal allowance parameter $\phi$ determined by the tax bracket $[\phi, \infty)$ establishes the after-tax residue $\pi(\xi, \tau) = (1-\tau) \cdot (\xi - \phi) + \phi$ of an alleged framed citizen's $\sigma = \xi$. Therefore, the dependency $\tau \equiv \tau(\xi, x)$ transforms $\pi(\xi, \tau)$ into fiscally realistic poverty line residue $\pi(\xi, \tau(\xi, x))$, representing the fiscal capabilities of the left-wing politicians. However, irrespective of the current expenditure on basic goods, the real cost of living does not necessarily match $\pi(\xi, \tau(\xi, x))$. Hence, ensuring realistic and fiscally idempotent rules and norms, and/or in particular attempting to avoid the $h$-effect of this mismatch or adopt rules to keep the effect tolerable at the least, an equation for a fiscally idempotent policy $\xi$ should be solved.

**Observation 1.** Constraint $u = \pi(\xi, \tau(\xi, x))$ is the necessary condition to uphold idempotent fiscal rules and norms of imposed budget-constraint $B(\xi) = x \cdot \tau \cdot W(\xi)$.

The observation claims that, whatever tax increase is implemented, the poverty line residue $u$ is unfeasibly high to be reached when the condition has been violated.

**Corollary.** When $u = \pi(\xi, \tau(\xi, x))$ is to be solved for $\xi$, the subsequent adjustments $\xi'$, $\xi''$, ... are unnecessary. For citizens, with incomes $\sigma < \xi$ or $\sigma > \xi$ to change their welfare positions, an option $\sigma$ is irrational—the root $\xi$ holds down (realistically limits) the $h$-effect. Appendix A3 includes all proofs.

The fiscally idempotent policies $\xi$ induce the basis for solutions in our game as fiscally idempotent compositions $[B(\xi), W(\xi)]$. A reasonable question thus emerges: Which taxable income $W(\xi)$ characterizes fiscally idempotent welfare policies $\xi$ for the delivery of LI subsidies $B(\xi)$? The answer is included in the following three constraints:

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4 Below, we continue to refer to the average taxable income as “wealth.”
Delivery-constraint by which the tax revenue is spent—the basic and public goods have been delivered. This form of constraint makes sense only for proportional or flat taxes. Flat taxes will later substantially simplify the method of function minimization with constraints.

\[ \tau \cdot W(\xi) = B(\xi) + g \]  

(1)

Budget-constraint imposed on LI subsidies finance in accordance with the slice \( x \) of the wealth-pie—the tax revenue. The left-wing politicians’ pledge to credit/debit the account \( B(\xi) \) that must be equal to the average of LI subsidies shifted by the policy \( \xi \).

\[ B(\xi) = x \cdot \tau \cdot W(\xi) \]  

(2)

Stability constraint that determines fiscally idempotent property of (2). In contrast to \( (\sigma, \tau) \in \mathbb{R}^2 \), we distinguish poverty line residues \( u = \pi(\xi, \tau) \) as one-dimensional curves \( \pi(\xi, \tau) \in \mathbb{R} \subset \mathbb{R}^2 \).

\[ u = (1 - \tau) \cdot (\xi - \phi) + \phi \]  

(3)

Taking the expression \( \tau(\xi, x) \equiv \frac{B(\xi)}{x \cdot W(\xi)} \) out of the constraint (2) and replacing \( \frac{B(\xi)}{x \cdot W(\xi)} \) into \( u = \pi(\xi, \tau(\xi, x)) \), the constraint given in (3) can be resolved with a fiscally idempotent policy for \( \xi \):

\[ L(\xi, x, u) = (\xi - \phi) \cdot B(\xi) - x \cdot (\xi - u) \cdot W(\xi) = 0. \]  

(4)

Referred to as the volatility-constraint, the constraint (4) determines the fiscal safety module. It holds down the \( h \)-effect amalgamating the constraints (3) and (2) by balancing the books accounting for LI subsidies.

**Summary.** The outcome \( \phi, \xi \Rightarrow z, x, \alpha, \tau, \langle u, g \rangle \) constitutes the taxpayers’ bargaining shield for income redistribution that relates to a bundle of variables or constants: \( \phi, \xi \) are controls, and \( z, x, \alpha, \tau \) are status variables \(^5\), while \( \langle u, g \rangle \) are the politicians’ competing proposals:

- \( \phi \) — the personal allowance establishing the tax bracket \( [\phi, \infty) \); it is an ex-ante control (tuning) variable, \( 0 < \phi = \text{const} < \xi \);
- \( \xi \) — the income frame, the poverty line; a policy determining who is living in poverty, as well as the choice or the control parameter;
- \( z \) — the size \( z = \tau \cdot W(\xi) \) of the wealth-pie; the account of tax revenue that equals public spending per capita when taxes are proportional;
- \( x \) — the slice of wealth-pie of size \( z \); a portion \( x \) of \( z \) to be deposited in favor of the left-wing politicians for funding the LI subsidies, \( 0 \leq x \leq 1 \);
- \( \alpha \) — the political power of the left-wing politicians, \( 0 < \alpha < 1 \);
- \( \tau \) — the marginal tax rate, the wealth-tax \( \tau(\xi, x) \) of the wealth amount \( W(\xi) \) determined by (1);
- \( u \) — the after-tax residue of the income frame equal to the poverty line \( \xi \), targets function \( u(\xi, x) \) of the left-wing politicians, as determined by (2) and (3);
- \( g \) — the targets function \( g(\xi, x) \) of the right-wing politicians, determined by (1) and (2); the account for the refund of public goods expenses per capita.

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\(^5\) Status and control variables are the prerogatives of control theory.
The slice $x$ and the marginal tax rate $\tau$, due to the constraints 1 through 3, become functions of variables $\xi, g : x \equiv x(\xi, g)$ and $\tau \equiv \tau(\xi, x(\xi, g))$. This form of dependence appears next in the module of alternating-offers bargaining.

5. Alternating-offers in the left-right political bargaining

Suppose that politicians, resulting in commitments of how to slice fairly the wealth-pie, agreed to play the alternating-offers bargaining game. In doing so, rational politicians are motivated to align the procedure to participate in any eventual agreement. The risk of a premature collapse during negotiations, especially early in the game, might be the driving force behind their commitment to reach the consensus. Once a consensus on slicing is reached, they must agree on who will make the decision about the size of the pie. Politicians negotiate on such matters when there are equal and symmetric preconditions in place that guarantee their equal rights. Thus, both will play an equal role in the decision regarding the size. However, assuming the welfare skills and competency shortage of right-wing politicians, a principal-agent problem may arise since $LWP$ and $RWP$ share non-conforming targets. In the spirit of these guidelines, while making welfare policy decisions, it will be realistic to reduce the scope of $RWP$’s duties, while allowing them to retain their advisory rights. We proceed as follows.

5.1. Bargaining procedure

We stressed that, in a representative democracy, slicing the tax revenue pie will always be subject to controversy. Recall that we consider two politicians—one acting in the role of $LWP$, who is struggling for basic goods, and the other, representing $RWP$, advocating for other public goods. A precondition for the bilateral agreement is that the targets of these two politicians depend solely on efficient policies of the $LWP$ within the framework of how to set the poverty line $\xi$. Thus, they are not particularly concerned with the size of the slice. As a consequence of this dependence, efficient poverty lines provide a fundamental correspondence to crucial slices. Accepting this precondition, the $RWP$ will only propose efficient lines. Failure to do so would result in all other slices being rejected with certainty. Nonetheless, it is realistic that the $RWP$ would—by negligence, mistake or some other reason—recommend an inefficient poverty line, which the $LWP$ would mistakenly accept. It is also possible that, in a reverse scenario, the $LWP$ would choose to disregard an efficient recommendation. This will be irrational handling as, in any agreement, regardless of the underlying motives, both politicians are committed by proposals to slices. Therefore, making a new proposal, the $LWP$’s recommendation on poverty lines makes a rational argument that the $LWP$ must accept or reject in a standard way. Such an account, instead of an agreement upon slices, as we believe, explains that the outcome of the bargaining game might be a desirable poverty line $\xi \in [\xi_1, \xi_2]$. Hereby, only the interval, referred to as the scope $[\xi_1, \xi_2]$ of negotiations, bids proposals, which now, by default, are binding efficient poverty lines with slices $x$. Consequently, the bargaining takes place exclusively in the interval $[\xi_1, \xi_2]$. Hence, $[\xi_1, \xi_2]$ is
the scope of RWP's efficient poverty lines of most trusted welfare policy platforms for negotiations, where players would choose poverty lines, accepting or rejecting the proposals. Political targets, depending on \([\xi_1, \xi_2]\), arrange the negotiators' contract curve \(S_b\) (shown in Figure 4 and 5) as a way to assemble the bargain portfolio. Given that the portfolio "has changed its color from slices to poverty lines," the politicians can now conceive themselves as making poverty line proposals. If a proposal is rejected, the roles of politicians change and a new proposal is submitted. The game continues in a traditional way, i.e., by alternating offers.

5.2. Analysis

We now proceed to a more accurate analysis of the situation. In accordance with the bargaining procedure, the left-wing politicians propose a slice \(x\) of the wealth-pie they commit to, and prescribe an efficient poverty line \(\xi\), which is fiscally reasonable to implement. If the right-wing politicians reject such a proposal, they will make an alternative proposal \(y\) of a slice of the pie they commit to, but can only recommend \(\xi\) as a desirable poverty line. Left-wing politicians, in general, are not committed to accept the recommendation \(\xi\). However, within the scope of negotiations \([\xi_1, \xi_2]\), the recommendation is the same as a proposal of the slice \(y\). This is the situation as, by rejecting the recommendation, the welfare position at the slice \(x = 1 - y\) that politicians have committed to will be inefficient—superfluous. The game continues, whereby each actor takes turns making a proposal, \(x\) or \(y\), with respect to the slice. In doing so, the politicians prescribe or recommend the poverty lines \(\xi\) in opposition to each other. When a rejection occurs, the momentary phase of the game consolidates a draft.

Although the rules can be perceived as fiscally idempotent, the game itself contains a new challenge. The elevated poverty line \(\xi\) does not necessarily increase the line residue \(u\), as the hazard (\(h\)-effect) is still present. The low-income citizens—the benefit recipients—can claim LI subsidies whereby an increased number of claims might have an effect that declines \(u(\xi, x)\). Indeed, in contrast to increasing poverty lines \(\xi\) and despite the required unavoidable increase in taxes, this will decrease the after-tax residue \(u(\xi, x)\). With the proviso that the left-wing politicians commit to the slice \(x\), the right-wing politicians are left with \(y = 1 - x\). Thus, the fiscally idempotent poverty line residues \(u(\xi, x)\) correspond to a more narrow set than \(0 \leq x \leq 1, 0 \leq y \leq 1\)—the set \(S_b\) of slices \((x, y)\), which arrange a contract curve, with poverty line \(\xi\) as a parameter.6

Assuming that the maximum of a single \(\cap\)-peaked curve can be reached, the peak position \(\xi^o = \arg \max \xi u(\xi, x^o)\) indicates an efficient welfare policy. Therefore, although the bargain portfolio of left-wing politicians contains an efficient policy \(\xi^o\) as a function of \(x^o\), the portfolio also contains the slice \(x = x^o\). Considering for any efficient policy \(\xi^o \in [\xi_1, \xi_2]\) the reverse situation, which corresponds to the top value given by \(u = u^o\), a unique slice \(x^o\) solves \(u(\xi^o, x) = u^o\) for \(x\); here,

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6 We stressed already the worsening quality of welfare services for all citizens when the LI-level is “climbing” high.
$g(\xi^o, x^o) = g^o$ represents the non-conforming targets of right-wing politicians. We can thus refer to the slice $x^o$ as an efficient slice matching the policy $\xi^o$. Depicted in Figure 4 in various projections (on targets $\{u^o, g^o\}$) and in Figure 5 on wealth amount $W$ and taxes $\tau$, efficient peaks $\xi^o$, which correspond to efficient slices $x^o$, arrange the geometry of what we refer to as the contract curve. This geometry highlights the top values $u^o$—namely, efficient policies of left-wing politicians at peaks $\xi^o$.

This corresponds to the well-known result of Canto et al., [7], 1981, p. 11—the Laffer curve:

"The marginal tax revenue raised decreases with increase in tax rates, finally reaching some point where the marginal tax revenue raised is zero. Beyond this point, any tax rate increases will reduce revenue collection."

Our result pertaining to the single-peaked targets of the left-wing politicians is similar. First, "poverty line residue $u$ being proposed in the normal range of LI parameter $\xi$." Next,

"…by passing through the top point of $u$ as a function, the proposals $u$ will be assessed and reviewed in the range of prohibited values of $\xi$.

We previously introduced an idempotent composition $[B(\xi), W(\xi)]$—the average $B(\xi)$ of the LI subsidies and the average $W(\xi)$ of the taxable income. The targets of the two politicians, reflecting their preferred rules and norms pertaining to LI subsidies, can now be set using the composition $[B(\xi), W(\xi)]$. At the end of the subsection, the composition will lead to an appropriately settled bargaining problem that will associate the threat originating from the implicit partaker—in the form of the electoral maneuvering of taxpayers—with an implicit risk of the negotiations collapsing prematurely. This requires adding two rigorous suppositions to the standard rules of the game we have already presented. Let us first specify the targets.

Targets of Politicians’ 1 and 2 and the third partaker’s implicit risk factor are defined as follows:

- Politician No.1, $u$ – the after-tax residue of the framed citizens with the income $\xi$ (the poverty line residue), basic necessities of the needy, cost of living;
- Politician No.2, $g$ – the account per capita of public goods expenses, targets that benefit all citizens in the society, expenses without LI subsidies;
- Third Partaker, $q, \tau$ – taxpayers’ electoral maneuvering facing higher taxes $\tau$ expressing an implicit risk $0 < q << 1$ of the negotiations collapsing prematurely.

Next, we assume that the rules and norms of the welfare arithmetic that are efficient with respect to the wealth pie slicing include the volatility-constraint (4), which certifies the idempotent composition $[B(\xi), W(\xi)]$ for the policy $\xi$. In the politicians’ game, the composition $[B(\xi), W(\xi)]$ could not be implemented without the volatility-constraint $L(\xi, x, u) = 0$ (Observation 1). This assumption is contingent on the conclusions of the previously undertaken analysis.

When varying $\xi$ under their own rules and norms, let us assume that $\text{LWP}$ propose a fiscally idempo-
tent policy }_{\xi} = \xi^o, which—for each slice }_x = x^o they commit to—binds }_x^o to }_{\xi}^o, ensuring, irrespective of who originates the proposals }_x^o or }_{y}^o, the efficient allocation of poverty line residue }_{u(\xi^o, x^o)} = \max_{\xi} u(\xi, x^o). Clearly, ineffective recommendation }_{\xi}, proposed by the RWP if }_\xi \neq \xi^o for slice }_{y}^o, will be rejected by the LWP. Consequently, an effective policy }_\xi = \xi^o must occur on contract curve amid efficient slices }_x^o at }_{u^o} = u(\xi^o, x^o), }_g^o = g(\xi^o, x^o) as an ongoing precondition for the agreement (this procedure was discussed previously). Indeed, LWP have no reason to reject efficient recommendation }_{\xi}, as doing so, when }_\xi \neq \xi^o, they cannot ultimately maintain the efficient commitment to }_x^o. Below, we drop the sign }_{^o} from all notations assuming the efficiency by default.

**Observation 2.** Idempotent policies }_\xi at the contract curve }_{S_b} = \langle u(\xi, x), g(\xi, x) \rangle, which certifies the composition }_{B(\xi), W(\xi)}], must satisfy the constraint

\[ D(\xi, x, u) = \frac{\partial}{\partial \xi} L(\xi, x, u) = \frac{\partial}{\partial \xi} \left[ (\xi - \phi) \cdot B(\xi) - x \cdot (\xi - u) \cdot W(\xi) \right] = 0. \]  

It is evident that poverty line residue }_u and the amount }_g of public goods at the contract curve }_{S_b} depend exclusively on policies }_\xi, }_{\langle u(\xi), g(\xi) \rangle} \in }_{S_b}. Recall that politicians are perceived as making proposals over policies }_\xi, rather than slice-proposals }_{(x, y)}. Contract curve }_{S_b} = u(g) in Figure 4 illustrates the targets. Particularly, when

\begin{align*}
Q(\xi, \tau, g) &= 0 \quad \text{Delivery (1);} \\
L(\xi, x, u) &= 0 \quad \text{Volatility (4);} \\
D(\xi, x, u) &= 0 \quad \text{Contract (5);}
\end{align*}

we have collated sub-expressions and have introduced some simplifications. These constraints—with the proviso of proportional (flat) taxes, together with the previously detailed preliminary settings }_{\tau^o_x > 0, \tau^o_x > 0, u^o_z < 0, u^o_z > 0, u^o_z < 0, u^o_z > 0, g^o_z > 0, g^o_z > 0, g^o_z \neq 0 lead to an analytical solution for poverty line }_{\xi}:

\[ u(\xi) = \xi - \frac{(\xi - \phi)}{v(\xi)}, \text{where } v(\xi) = 1 + (\xi - \phi) \cdot \left( \frac{B'(\xi)}{B(\xi)} - \frac{W'(\xi)}{W(\xi)} \right); \tau(\xi) = \frac{1}{v(\xi)}. \]

\[ g(\xi) = \frac{W(\xi)}{v(\xi)} - B(\xi); \text{the size of wealth-pie } z(\xi) = B(\xi) + g(\xi) = \frac{W(\xi)}{v(\xi)}. \]

The targets functions }_g(\xi) and }_u(\xi) in the form presented above are, in fact, not a subject to any constraints. They are mathematically derived in Appendix A4.

\[ \pm \text{ rates } W'(\xi) \leq 0, W'(\xi) \geq 0 \text{ of the changes in the wealth amounts } W(\xi) \text{ are essential for the analysis, whereas the function } B(\xi) \text{ is valid only when } B'(\xi) > 0, \text{ and } 0 < \phi < u < \xi. \]
The bargain portfolio contains the scope \([\xi_1, \xi_2]\) of negotiations that politicians will follow in the traditional way. Now, the left-wing politicians propose \(\xi \in [\xi_1, \xi_2]\), which the right-wing politicians either accept or reject. In the latter situation, the roles of the two actors change and, under the alternating-offers rule, it is the turn for the right-wing politicians to make a proposal—recommendation. However, they will only make, from left-wing politicians view, an effective proposal \(\xi\). The game continues, with the two politicians changing roles, until the proposal \(\xi\) is accepted. The policy \(\xi\) on poverty is a control parameter of the welfare arithmetic rules and norms. When the proposals over slices \((x, y)\) of the wealth-pie in eventual agreement are being negotiated, the efficient policy \(\xi\) is referred to as a proposal of policy on poverty.

Before proceeding with this line of analysis, let us recall the threat phenomenon created by taxpayers that increases the implicit risk of the negotiations collapsing prematurely. As noted previously, if politicians reject their counterpart’s proposal—knowing that it is risky to continue the bargain—they will likely consolidate a draft. Taxpayers might emanate a threat to vote against the draft when politicians, without fulfilling the taxpayers’ terms, try to continue bargaining over excessively costly proposals, thereby putting the negotiations at risk of collapse, see above.

Suppose that politicians bargain over all fiscally idempotent policies \(\xi \in [\xi_1, \xi_2]\) within the scope of negotiations \([\xi_1, \xi_2]\). Below, we follow the alternating-offers game \(\Gamma(q)\) with an exogenous risk \(0 < q < 1\) of a premature collapse, well described by Osborne & Rubinstein, [30], 1990, pp. 71-76. We posit that, each time the proposal \(\xi\) is rejected by one of the politicians, the momentary phase of the game consolidates a draft, which can be opposed by the taxpayers, as previously mentioned. In these circumstances, the politicians might be uncertain on how to proceed, if the taxpayers' terms are not met. As a result, they might choose to leave the bargaining table prematurely. The outcome \(\langle u_1, g_1 \rangle = \langle u(\xi_1), g(\xi_1) \rangle, \langle u_2, g_2 \rangle = \langle u(\xi_2), g(\xi_2) \rangle\), extracted from the endpoints \(\xi_1 < \xi_2\) of contract curve \(S_b\), is the worst-case scenario, which naturalizes this risk \(q\).

What is known as the "well-defined bargaining problem," first introduced by Roth, [37], 1977, or the individual rationality associated with the Nash [27] bargaining scheme \(\langle S, d \rangle\), seems to be instructive to look closer. Indeed, inequalities \(g_1 > g_2, u_1 < u_2\) hold for the pair \(d = \langle d_1 = u_1, d_2 = g_2 \rangle\). Synthesizing the politicians’ unfavorable outcome \(\{u_1, g_1\}, \{u_2, g_2\}\) into a policy \(\delta\) on poverty introduced below will naturalize the Nash disagreement point \(d\) into the problem \(\langle S_b, d \rangle, S_b \subset R^2\). Thus, compared to the traditional approach of compact convex set \(S \subset R^2\), inequalities \(s > d\) are also true for any pair \(s \in S_b\). Therefore, the pair \(\langle S_b, d \rangle\) for the contract curve \(S_b\) becomes a well-defined bargaining problem. However, as whether the policy \(\delta\) is a fiscally idempotent outcome of the game is not immediately apparent, the following observation removes any doubt.
**Observation 3.** To test whether the point \( d = (d_1,d_2) = (u,z,g) \) becomes a fiscally idempotent outcome of the left-right political bargaining, it is necessary and sufficient that there exists a policy \( \delta \) on poverty, which solves the equation:

\[
(\delta - \phi) \cdot (B(\delta) + d_2) - (\delta - d_1) \cdot W(\delta) = 0;
\]

The condition \( \delta \notin [\xi_1,\xi_2] \) must hold true.

It should be noted that, in the worst-case \( \delta \), the average wealth amount redistributed in the society is \( W(\delta) \) and the average of expenses for funding the LI subsidies is \( B(\delta) \). The proposal \( \delta \) depends on the endpoints of the bargaining interval \([\xi_1,\xi_2]\). This dependence, provided that Equation (6) can be solved for \( \delta \), serves as the basis for validation of the pre-equity condition of breakdown, as discussed in Section 7.

**Observation 4.** In the alternating-offers game \( \Gamma(q) \) with the risk \( 0 < q < 1 \) of negotiations collapsing prematurely into the disagreement point \( (d_1,d_2) \), the functions \( (u(\xi) - d_1)^a \) and \( (g(\xi) - d_2)^{1-a} \) imply bargaining targets of left- and right-wing politicians respectively. The solution \( \lambda \) of the well-defined bargaining problem \( \langle S_b,d \rangle \) is close to the pair \( (\lambda_1,\lambda_2) \), \( \lambda_1 \leq \lambda \leq \lambda_2 \), thus solving the equations \( (1-q) \cdot (u(\lambda_1) - d_1)^a = (u(\lambda_2) - d_1)^a \) and \( (1-q) \cdot (g(\lambda_2) - d_2)^{1-a} = (g(\lambda_1) - d_2)^{1-a} \) for variables \( \lambda_1,\lambda_2 \) (without proof).

As explained by Osborn & Rubinstein, [30], 1990, p. 75, in our case study, the outcome of this bargaining game \( \Gamma(q) \) will encapsulate the power indicators \( (\alpha,1-\alpha) \) of the left- and right-wing politicians. In the next section, we consider political design of power indicators \( (\alpha,1-\alpha) \) using the solution \( \lambda \) minimizing the tax burden with respect to an appropriately settled bargaining problem \( \langle S_b,d \rangle \).

6. Political and electoral maneuvering

The issue of avoiding the risk \( q > 0 \) of premature collapse of political outmaneuvers has not yet been addressed. Clearly, from the voters' perspective, the policy that minimizes tax is always the most desirable choice. However, despite knowing voters' negative attitude toward higher taxes, the minimum tax might not necessarily be a desirable outcome in politicians' view. Thus, politicians may choose to disregard the voters' wants because political power of LWP or RWP, as rational actors/politicians, might be strong enough to negotiate selfish decisions favorable for both politicians alone. In order to outmaneuver politicians and prevent them from making selfish decisions, resulting in ultimate collapse in the negotiation process, their political power indicators \( (\alpha,1-\alpha) \) are also considered. More specifically, \( 0 < \alpha < 1 \), where \( \alpha \) signifies LWP’s political power, and \( 1-\alpha \) signifies the political power of RWP, is included in the analysis.
We will elaborate on power indicator $\alpha$ further using an illustrative example based on the previously discussed axiomatic bargaining. Considering the $\arg\max$ formula of $f(x,y,\alpha)$, a question emerges: What is the standard that will assist the players when designing power indicator $\alpha$ of the 1st actor? What will facilitate the 1st actor during to negotiations in obtaining a desired, or any other (e.g., $\frac{1}{2}$) slice of the pie? To answer these questions, let us assume that the 2nd actor might only accept or reject the proposal. We can start redesigning the power indicator $\alpha$ by replacing $u(x)$ with $x, y = 1 - x, g(y)$ with $\sqrt{1 - x}$, and taking the derivative of the result $f(x, 1 - x, \alpha)$ with respect to the variable $x$ by evaluating $f'_x(x, 1 - x, \alpha)$. Finally, suppose for a moment that $\frac{1}{2}$ of the pie is an allegedly desirable solution. With $x = \frac{1}{2}$, the equation $f'_x(x, 1 - x, \alpha) = 0$ can be solved for $\alpha$; indeed, the equation $f'_x(\frac{1}{2}, \frac{1}{2}, \alpha) = 0$ resolves for $\alpha = 1/3$. A technical person can shed light on the solution.

In general, one might feel comfort in the following egalitarian judgment:

To count on the half of the pie is a realistic attitude toward the 1st actor’s position of negotiations. Indeed, sooner rather than later, even in the face of the fact that the 2nd actor is twice as tough a negotiator, the 1st actor might predict whatever she/he—the 2nd actor—prefers. Therefore, the former would have the latter agree to a concession.

When we attempt to redesign accordingly political power indicators to $(\alpha, 1 - \alpha)$, we assume that politicians will try to slice the wealth-pie in a similar manner. In doing so, we suppose that both politicians are ready to proceed with tax concessions. Reflecting just illustrated axiomatic bargaining toward allegedly desirable half of the pie, we proceed as follows.

In accordance with our analytical solution without constraints, the contract curve $S_b = u(g)$ corresponds to a curve $(u(\xi), g(\xi))$. Moving along the curve while combining the movement along the scope of negotiations $[\xi_1, \xi_2]$, the voters’ emphases $\tau(\xi)$ lead to detection of $\tau(\xi)$ with $\tau_{\min}$:

$$
\min_{\xi \in [\xi_1, \xi_2]} \tau(\xi) \bigg| \tau(\xi) = \frac{1}{v(\xi)}.
$$

With the proviso that $\tau(\xi)$ is concave and smooth enough, the detection point of $\tau_{\min}$ is the root $\lambda$ of the equation $\tau'(\xi) = 0$. Consequently, akin to egalitarian judgment above, the root $\lambda$ might help in redesigning of the rules and norms of the welfare arithmetic. This can be done, adjusting the $\alpha$, in a way that the political power $\alpha$ of the left-wing politicians will be sufficient to persuade the right-wing politicians to agree upon the poverty line residue $u(\lambda)$.

Indeed, in the left-right political bargaining, the old standard (discussed above) of how to slice the pie can now be a new redesigned standard pertaining to how to plan the welfare arithmetic rules and norms. Under this premise, we can set $f(\xi, \alpha) = (u(\xi) - d_1)^\alpha \cdot (g(\xi) - d_1)^{1-\alpha}$, where $\alpha$ facilitates the political power of the LWP. Once again, but now instead of $x = \frac{1}{2}$, we suppose that $\xi = \lambda$ is an allegedly desirable solution. Similarly, we first take the derivative of $f(\xi, \alpha)$, with respect to $\xi$ evaluating $f'_\xi(\xi, \alpha)$, and then we solve the equation $f'_\xi(\xi, \alpha) = 0$ for $\alpha$. As a result, the root $\alpha$ will correspond to the redesigned political power of the left-wing politicians. This is the result as it appears.
Summary. To control the left- and right-wing politicians’ agreement on allotment \((x, y)\) slicing the pie, the implicit partaker—the taxpayers in the result as it appeared above—can accept or reject a premature agreement archived at the moment during the negotiations, thereby voting for or against the slicing. The taxpayers will always favor the policy \(\lambda\) that minimizes the tax burden. This restriction allows us to rebalance the welfare institutions by appropriate design of power indicator \(\alpha\) of the left-wing politicians ensuring that the most favorable allotment \((x, y)\) of the wealth-pie would incorporate the Nash axiomatic—the minimum tax—solution \(\lambda\) into the bargain portfolio as the best outcome. This is our case study of tax policy in which only few would object to a proposal that corresponds to the marginal wealth-tax minimum at the contract curve. In doing so, the implicit pressure of voters will be lower. To be implemented in favor of all citizens, the minimum appears to be a desirable consensus.

Observation 5. Given that politicians can reach a preliminary agreement on wealth-tax \(\tau = \tau(\xi)\), condition \(\lambda = \arg\min_{\xi \in [\xi_1, \xi_2]} \tau(\xi)\) is necessary to put forward a poverty proposal \(\lambda\) before taxpayers by appropriately designing the power indicator \(\alpha\) in advance. At the contract curve \(S_b\), the proposal \(\lambda\) outlines a unique outcome \(\phi, \xi \Rightarrow z, x, \alpha, \tau(\lambda), \{u(\lambda), g(\lambda)\} \in S_b\).

7. Discussion

One possible way to reveal the true essence of the economic reality behind the left-right political bargaining could be determining whether it is true that funding LI subsidies and maintaining the budget in balance will be difficult to sustain when the tax burden for all citizens is decreasing. On the surface, it seems that, at some point, fairness and equity might no longer be the main requirement because the "rich simply get richer and the poor get poorer." Indeed, the effect of "tax relief for the rich" seems to affect the well-being of less fortunate citizens adversely. In the face of these controversies, no one can estimate the extent of potential fallout that might result from such outcomes. As a taxpayer, the reader is invited to contribute to this analysis and attempt to answer this dilemma. The citizens are those that should ultimately decide what needs to be done in order to socially plan and redesign the welfare arithmetic rules and norms. Taking advantage of this opportunity, it is instructive to perform an exercise related to the most appropriate choice of welfare policy, as shown in the "minimizing wealth tax" column of Table 1. We estimated that tax relief for all citizens—despite minimizing the wealth tax—is, in fact, fiscally safe, while also ensuring just and fair income redistribution for all citizens.

The following discussion deserves, perhaps, some guidance, due to the assumptions made during the analysis. Before commenting on those, it is worth noting that the case study presented here should be understood as purely normative—namely, "what ought to be" in economic or political matters, as opposed to "what is." Therefore, despite the fact that, in the preceding analysis, no actual situation was presented, our theoretical results rest on the assumptions delineated below.

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8 The relationship between an axiomatic and the one that offers the alternating bargaining solution is well known from Osborn and Rubinstein, [18], 1990, p. 75.

9 Table 1 was created by numerical simulation carried out upon imaginary income redistribution of citizens.
First, we worked under the premise that politicians only made promises that can be fulfilled—fiscally save proposals. However, fiscal safety, taken separately, even when attempted in accordance with the rules and norms in force, could lead to unjust and unfair solutions. Indeed, taken at will, fiscal safety might be a profoundly mistaken idea of justice. In Table 1, we present the percentage of citizens below the poverty line, thus establishing the poverty rate.\(^\text{10}\) Driven at will, the official poverty rate, in accordance with the disagreement column in Table 1, could cause the poverty rate to decline below 0.41%, which wrongly appears to be the most just and fair.

Second, the wealth-pie redistribution compensated for the inequalities in the income of citizens that were below the poverty line. Usually, similar parameters are in the national government competence. While taking into account increases in the cost of living, the official number of individuals living in poverty should be adjusted annually according to government guidelines. Although our key assumption was that the right-wing politicians inherited no more than an advisory authority, the rules and norms that govern the poverty line determination have been solely under the mandate of the left-wing politicians. This decision was made because, in the analysis, we deliberately emphasized the distinctions between stereotypical motivations of left- and right-politicians. In our view, welfare protection that is most likely to be just as fair should be addressed as an independent institute, or better yet, as an assembly of independent institutes or legal charity foundations. We believe that, in our case study of organizational independence, welfare protection could be expected to yield effective welfare policies. Thus, in determining an efficient policy on poverty, we concluded that left-wing politicians should be in a privileged position that allows them to prescribe the poverty line independently. Only when these guidelines of independence are applied, the value judgment based upon the data presented in Table 1 makes sense. Still, it should be noted that the characterization of whether setting up such a privilege was a positive or negative restriction requires additional investigation.

Next, we focused on the political power indicator \(\alpha\), which highlights the amount of resources, skills and competence of left-wing politicians. The fundamental factor in our analysis was the welfare protection of the society as a whole to justify and maintain welfare duties under the principle of how the state ought to act when trying to fulfill its welfare mission. When the decision made by the politicians is not in line with the wants of special interest groups, as previously pointed out, welfare protection could be a never-ending theme in political debates and election campaigns, and a source of significant political competition. A conflict of political interests might lead to violent upsets, providing the opportunity to develop policy in favor of these groups. According to the foregoing account, which requires considerable administrative efforts and fiscally unrealistic expenses—and previous observations pertaining to the independence of the welfare services—we believe that there is no reason for the taxpayers to have sophisticated left-wing institutions. Thus, recognizing the vital role of the right-wing politi-

\(^{10}\) Poverty rate determines the percent of anyone who lives with income less than official poverty line. The poverty line separates the rich (those with an income higher than the line), from the less fortunate (with an income below the line).
cians, due to their central position to decide who will be purchasing and delivering public goods, in the interpretation of the parameter $\alpha$, we believed that it was beneficial to impose a lower grade $\alpha$ to the left-wing politicians, with a corresponding higher grade $1-\alpha$, $0 < \alpha < 1$, assigned to the right-wing politicians. Thus, it was reasonable to assume that left-wing politicians, with almost no extra effort, would demonstrate an ample degree of readiness to make efficient decisions. Herewith, in planning and adjusting the size of the pie to suit a fiscally realistic welfare policy to settle and assist the state welfare mission, we tried to redesign the balance of political power between the left- and right-wing politicians by adjusting the power indicator $\alpha$ imposed on the former. This enabled us, in order to benefit all citizens in society, in our view, to adjust the state rules and norms of the welfare arithmetic, aligning it closer to the legal responsibilities and moral obligations of the taxpayers. We called the process of adjusting the power indicator $\alpha$ a political design.

The design of political power indicator $\alpha$ is a difficult and time-consuming negotiation process. As prolonged negotiations might not be in the interest of anyone, taxpayers might not pursue it, even if the balance of political power can be ultimately reached. Therefore, we supposed that electoral maneuvering of voters against higher taxes might put the negotiations at risk of a premature collapse. In particular, it was deemed acceptable to assume presence of an implicit risk of taxpayers defecting to the other side, which could interrupt negotiations ahead of the schedule. Thus, we brought the problem of likelihood of negotiations collapsing into focus. In our case study that the failure of negotiations was extremely undesirable for both politicians, we hoped that this would be an incentive to move toward a solution faster. Alternatively, they would be more motivated to agree on terms of a contract, where both sides approach each other by making considerable tax concessions. In the view of welfare receipt of subsidies, a policy of higher tax rates might be the most favorable and just solution. Yet, from the consumer's perspective, the minimum tax rate always lies first at citizens' wants. Therefore, for those citizens who finance the LI subsidies, as we assumed in the analysis, the minimum tax rate provided a more just and fair redistribution of wealth. The minimum tax rate also provided an outcome $\lambda$ in which the designed power indicator $\alpha$ visualize the society's common denominator. Assuming, as we previously did, in accordance with the rules of the game, that outcome $\lambda$ minimizing taxes could be politically designed, the outcome $\lambda$ provided insight into what policy should entail. Such a politically designed outcome was, as we supposed, worth the time and effort, even if the vision was a utopia.

Table 1, presenting all four assumptions, suggests several proposals for taxpayers to vote on. Note that, when voting for policy of equal left-right wing political power, the policy $\eta = 79.23$, is less just and less fair than the outcome $\lambda = 45.50$, where the minimum 26.52% of marginal tax rate is reached. Thus, only policy/outcome $\lambda$ on the poverty line (Figure 5) can be the only desirable political consent. Indeed, in the variety of rules in the game the left- and right-wing politicians play, when engaged in an interaction aimed at implementing equal/egalitarian policy $\eta$, the equal political power $\alpha = 0.5$ of the LWP was stronger than 0.21. However, consumers' goal can still be achieved by applying the weaker policy $\lambda = 45.50$ for the tax rate $26.52% < 28.21%$, although the outcome of the weakened political
power indicator $\alpha = 0.21$ is yet to be confirmed. Therefore, through a reduction of taxpayers’ obligations—even with LWP’s weakened political position—the LWP will be able to come to a desirable agreement with the RWP maintaining the most just and fair poverty line of wealth for all citizens.

In closing the discussion, we would like to point to a decision $\delta$ that corresponds to the breakdown of negotiations. Utopian society, planned according to the event of a breakdown, as shown in Table 1, seemingly ignores welfare protection because practically all citizens are considered rich by default. In other words, poverty does not exist. Given this utopian society, financing expenses almost entirely with respect to vital public/non-basic goods, the breakdown policy $\delta$, on equity condition, requires $-2.49$ public debt per capita. This, in turn, will require borrowing or money printing, e.g., reducing poverty through natural assets for refunding the debt. We, therefore, admit that, based on the lowest tax burden of 26.52%, a self-financing tax system has a better chance of being implemented.

8. Concluding remarks

This case study has contributed important knowledge to the field. We followed the delivery of basic goods under the LI rules and norms of welfare arithmetic. By negotiating the terms along the edge of the poverty line that treated all citizens equally, the politicians representing the opposing sides of the bargain decided to whom the basic goods should be distributed. The expenses pertaining to basic goods, as well as those associated with public (non-basic) goods, were separately estimated by transforming the expenses into targets functions of the poverty line. Based on the analysis, we concluded that an elevated poverty line, as a parameter, gave rise to inverse working incentives of benefits claimants, referred to as the hazard or $h$-factor effect. This resulted in unbalanced books accounting for the delivery of basic and non-basic goods to their respective destinations. For this reason, contra $h$-factor, the balance became crucial in resolving the political conflict between left- and right-wing politicians, as the two key actors in the bargaining game.

Given the conflicting interests of the left-right wing politicians, and the need to resolve the welfare policy dilemma, both actors were willing to make tax concessions. The root of the controversy was that, in pursuing their own political causes, the left-wing politicians struggled for the increase of basic goods, whereas the right-wing politicians—in response to public wants—advocated for meeting the needs for non-basic goods. Left-wing politicians gave credit to the tax system to guarantee a reasonably high living standard for benefit claimants. However, both politicians were aware of the taxpayers’ electoral maneuvering in favor of lower taxes, which could put the negotiations at risk of premature collapse. This threat was the only driving force in reaching the consensus. We argued that political arguments demanding higher taxes were weak, as overly costly proposals must subsidize the excessive number of benefits claimants, which, in spite of the tax increase, could lead to diminished quality of the welfare services. In turn, the excessive number of claims could generate further requests for additional financial support through tax channels. In order to satisfy the wants of those who did not bear the additional costs, and who could only approve the request on the terms of fiscally safe welfare policies, we reduced the scope of negotiations to the fiscally reasonable domain of political targets.
In view of the above, a pretext for the analysis of the domain and the extent of bargain portfolio of two visionary politicians, denoted as LWP and RWP, were established. The portfolio was supposed to account for politicians having non-conforming targets. Instead of slicing the wealth-pie, such an account allowed for the inclusion a guide on how the eventual consensus ought to be analyzed and interpreted within the scope of negotiations $[\xi_1, \xi_2]$ at the contract curve. In this context, the left-right wing political power indicators, specified by the bargaining problem solution, were supposed to be politically designed in advance to tailor them in accordance with the taxpayers' visions and ambitions.

It was initially deemed that, due to the uncertainty in the selection of the breakdown policy, we could only treat the left-right wing political power as indicators given exogenously. However, a condition—at least true in the valuable examples presented here—that can encode the indicators endogenously was found and named "the pre-equity of breakdown."

References


Appendices

A1. Example and Illustration

We proceed with a specific allocation of the welfare policy encapsulating samples of income density distribution, parameterized by poverty line $\xi$, similar to an exponential function:

$$P(\sigma, \theta + h \cdot \xi) = \frac{1}{(\theta + h \cdot \xi) \cdot \Gamma(m)} \cdot \exp\left( -\frac{\sigma}{\theta + h \cdot \xi} \right),$$

where $\theta = 61.9$, $m = 2.07$, and $h = -0.18$ are additional ex-ante parameters. More specifically, $\theta$ controls the wealth of citizens—a horizontal shift of samples; $m$ controls inequality—a vertical shift; $h$ is a hazard parameter; and $\Gamma(m)$ is an extension of $(m-1)!$ to real numbers.

The density function $P(\sigma, \theta + h \cdot \xi)$, depending on $\xi$, reflects the initial redistribution of wealth through tax channels. Moreover, political decision $\xi' > \xi$ shifts the distribution $P(\sigma, \theta + h \cdot \xi')$ horizontally toward the allocation $P(\sigma, \theta + h \cdot \xi)$ that favors less wealthy. When shifted, the distribution $P(\sigma, \theta)$ masks the $h$-factor, $h = 0$, of the benefit claimants. The rate of change $Hz(\xi) = h \cdot \dot{\alpha}(\theta + h \cdot \xi) < 0$ of the policy $\xi$ quantifies a fiscally tolerable hazard ($h < 0$).

Figure 3. At the sample $P(\sigma, \theta + h \cdot \frac{1}{2} \mu)$ of the income density distribution, $\mu$ solves the equation $\int_{0}^{\xi} P(\sigma, \theta + h \cdot x) \, d\sigma = 0.5$ for $x$; $\mu = 82.30$.

The sample $\xi = \frac{1}{2} \mu$ (median income = $\mu$) can be presented as Lorenz Curve, where citizens below an income 151.48, i.e., 75% of the population, have 51.11% of a total cumulative income, while the remaining 25%, with incomes at or above 151.48, have 48.89%. Gini Coefficient equals 0.37. Horizontal shifts do not affect the Gini coefficient, while the vertical ones do. A more detailed example is available upon request.
A2. Simulation

In order to perform simulations, the expressions for average $B(\xi)$ of expenses on the LI subsidies and average taxable income—the wealth amount $W(\xi)$—can incorporate income density distribution $P(\sigma, \theta + h \cdot \xi)$ in a more realistic but general form:

$$B(\xi) = r \cdot \int_0^\xi (\xi - \sigma) \cdot P(\sigma, \theta + h \cdot \xi) \, d\sigma; \quad r \cdot (\xi - \sigma)$$

is the LI subsidy, $0 < r < 1$;

$$W(\xi) = \int_0^\xi (\sigma + r \cdot (\xi - \sigma) - \phi) \cdot P(\sigma, \theta + h \cdot \xi) \, d\sigma + \int_\xi^\sigma (\xi - \phi) \cdot P(\sigma, \theta + h \cdot \xi) \, d\sigma, \quad 12$$

The taxation of the total income $\sigma + r \cdot (\xi - \sigma)$ of the needy is in compliance with the rules and norms in force; the $h$-factor reveals the inverse working incentives—namely, the feedback of the welfare recipients. These rules and norms are in line with our amendment to Friedman [14] proposals.

At this point, it is useful to verify that a disagreement policy $\delta$ under the primacy of equity principle of breakdown might be an outcome of the game. There is no reason why the equation $(\delta - \phi) \cdot (B(\delta) + d_z) - (\delta - d_z) \cdot W(\delta) = 0$, in accordance with the observation 3, should have a solution in general. However, for the income density $P(\sigma, \theta + h \cdot \xi)$ (see above), a solution can be found. Given monetary targets $(u, g)$ at the endpoints $(u_1 = 6.44, g_1 = 47.18, u_2 = 89.26, g_2 = -2.49)$ of the scope of negotiations—within the interval $[\xi_1 = 8.00, \xi_2 = 144.54]$—one can discover that the pair $d = (d_1 = u_1, d_2 = g_2) = (6.44, -2.49)$, $u_1 < u_2$, $g_1 > g_2$ consolidates an equity for breakdown policy $\delta = 6.39 \in [\xi_1, \xi_2]$; wealth $W^* = 120.46$ and tax $\tau^* = -2.06\%$.\(^{13}\)

\(^{12}\) In the left-right political bargaining, the choice of $\xi$, in general, is also an issue of the average income $a(\theta + h \cdot \xi)$ maintenance to uphold $a(\theta + h \cdot \xi) > W(\xi)$ within the “striking” distance from $W(\xi)$, which can be ensured through proper choice of the personal allowance constant $\phi > 0$, where $\phi$ identifies a flat tax bracket $[\phi, \infty)$. The average $a(\theta + h \cdot \xi)$ of income $\sigma$ over the density sample $P(\sigma, \theta + h \cdot \xi)$ equals

$$\int_0^\infty \sigma \cdot P(\sigma, \theta + h \cdot \xi) \, d\sigma.$$

\(^{13}\) It should not be surprising that the amounts of public goods and tax rates may be negative. Ensuring this outcome of the game, the interpretation suggests that the simulated breakdown demonstrates a specific target deficit on public goods when it is impossible to cover the costs through taxes. In such a scenario, as we have pointed out earlier, when discussing the breakdown, the solution is resorting to an external loan, money printing, or use of natural resources, if the latter are available.
The magnitude and dimension of poverty proposals to be debated or implemented, as "outcomes of the left-right political bargaining," are given in the following table.

Recall already known proposals for incomes $\eta, \lambda_1, \lambda, \lambda_2, \delta$ (whereby $\delta$ is outside of the scope of negotiations—$\delta \notin [\xi_1, \xi_2]$); and the poverty proposal $\frac{\mu}{2}$, as follows:

- $\eta$: the policy on poverty with equal left-right wing political power; the left- and right-wing political organizations are in symmetrical positions or in equal roles;
- $\lambda_1$: the outcome of the alternating-offer game—what the right-wing politicians accept;
- $\lambda$: the policy on poverty minimizing wealth-tax;
- $\frac{\mu}{2}$: 50% of the median income, indicating that half of the population earn income above $\mu$, with the income of the remaining half below $\mu$;
- $\lambda_2$: the outcome of the alternating-offer game—what the left-wing politicians accept;
- $\delta$: the least desirable outcome, resulting in the breakdown policy or disagreement, which naturalizes the risk of negotiations' premature collapse, caused, for instance, by mutual traps.

Table 1. Numerical simulation behind the left-right political bargaining:

<table>
<thead>
<tr>
<th>Obtained by means of income density distribution (Figure 3); personal allowance $\phi = 4.03$; $\theta = 61.9; h = -0.18; m = 2.07$; $r = \frac{\mu}{2}$: a proportion to $(\xi - \sigma)$</th>
<th>Policy of equal—symmetric political power</th>
<th>LWP proposal accepted by RWP</th>
<th>Proposal minimizing wealth tax</th>
<th>Poverty line = 50% of median income</th>
<th>RWP proposal accepted by LWP</th>
<th>Policy of disagreement, the breakdown</th>
</tr>
</thead>
<tbody>
<tr>
<td>Poverty line; welfare policy $\xi$</td>
<td>$\eta$</td>
<td>$\lambda_2, q = 5%$</td>
<td>$\lambda, q \approx 0%$</td>
<td>$\frac{\mu}{2}$</td>
<td>$\lambda_2, q = 5%$</td>
<td>$\delta$</td>
</tr>
<tr>
<td>79.23</td>
<td>40.79</td>
<td>45.50</td>
<td>41.15</td>
<td>50.28</td>
<td>6.39</td>
<td></td>
</tr>
<tr>
<td>Poverty rate: percentage of citizens below the poverty line</td>
<td>47.36%</td>
<td>15.73%</td>
<td>19.15%</td>
<td>15.99%</td>
<td>22.81%</td>
<td>0.41%</td>
</tr>
<tr>
<td>Political power of left-wing politicians $\alpha(\xi)$</td>
<td>0.50</td>
<td>0.18</td>
<td>0.21</td>
<td>0.18</td>
<td>0.24</td>
<td>Not defined</td>
</tr>
<tr>
<td>LI netto; the after-tax residue of $\xi$ $u(\xi)$</td>
<td>58.02</td>
<td>31.02</td>
<td>34.50</td>
<td>31.29</td>
<td>37.99</td>
<td>6.44</td>
</tr>
<tr>
<td>Account for public goods expenses $g(\xi)$</td>
<td>19.02</td>
<td>27.63</td>
<td>26.70</td>
<td>27.56</td>
<td>25.75</td>
<td>-2.49</td>
</tr>
<tr>
<td>Account for LI subsidies transfers $B(\xi)$</td>
<td>10.61</td>
<td>1.57</td>
<td>2.17</td>
<td>1.62</td>
<td>2.91</td>
<td>0.01</td>
</tr>
<tr>
<td>Account for public spending, the size of the wealth-pie $z(\xi)$</td>
<td>29.63</td>
<td>29.20</td>
<td>28.87</td>
<td>29.18</td>
<td>28.66</td>
<td>-2.48</td>
</tr>
<tr>
<td>Average taxable income—the wealth amount $W(\xi)$</td>
<td>105.04</td>
<td>109.95</td>
<td>108.86</td>
<td>109.87</td>
<td>107.88</td>
<td>120.46</td>
</tr>
<tr>
<td>Wealth-tax, marginal tax rate $\tau(\xi)$</td>
<td>28.21%</td>
<td>26.56%</td>
<td>26.52%</td>
<td>26.56%</td>
<td>26.56%</td>
<td>-2.06%</td>
</tr>
</tbody>
</table>
Figure 4. The monetary targets of left- and right-wing politicians are depicted on the vertical and horizontal axes, respectively. The graph shows the contract curve sloping from $\xi_2$ toward $\xi_1$, projected on the surface of basic goods contra vital goods—the projection of efficient poverty lines $\xi$ resolving the contract constraint (5).

Figure 5. The graph depicts two different motions for a vote: for the higher tax $\tau = 29.01\%$, marked by the horizontal line, and the lowest tax $\tau = 26.52\%$, marked by the vertical dash. Indicated by $\rightarrow$, at cross points of the contract curve with the horizontal line, we observe conflicting emphases of the voters. The allotments of lower basic but higher public goods are shown on the left, while this distribution reverses towards the right side of the graph, as the allotments of basic goods increase while those for public goods decrease. Thus, the higher tax $\tau = 29.01\%$ cannot be a political consent, Observation 5.

A3. Verification
**Proof of observation 1.** Let us now assume an inverse scenario, whereby \( u > u' = \pi(\xi, \tau(\xi,x)) \). Here, the left-wing politicians—\( \text{LWP} \)—aim to improve the poverty line residue \( u' \), i.e., an after-tax residue of a framed citizen \( \sigma = \xi \) with income equal to the poverty line \( \xi \). By initiating a new rule for policy \( \xi' > \xi \), the \( \text{LWP} \) attempt to implement \( u > u' \). Because of the inequalities \( u \geq \pi(\sigma, \tau(\xi,x)) > u' \), for some highly pragmatic benefit claimants \( \sigma \), it becomes apparent that they can be better off by claiming LI subsidies. Consequently, actions of these claimants will increase the expenditure \( B(\xi') = \tau(\xi,x) \cdot W(\xi) \) to deficit \( B(\xi') > x \cdot \tau(\xi,x) \cdot W(\xi) \). The balance was valid in the past, when \( \tau(\xi,x) \equiv \frac{B(\xi)}{x \cdot W(\xi)} \).

Thus, the only option that would ensure that the balance in maintained, as the \( \text{LWP} \) must stay committed to \( x \), is to adjust \( \tau(\xi,x) \) to \( \tau(\xi,\xi',x) = \frac{B(\xi')}{x \cdot W(\xi)} > \tau(\xi,x) \). Thus, \( x \) was fixed by the agreement. Otherwise, keeping the old policy \( \xi \) intact, the \( \text{LWP} \) could—through a decrease in \( x \)—violate the commitment \( x \). However, as they cannot directly change \( x \), they resort to reducing the deficit by tax increase. If \( u > \pi(\xi', \tau(\xi,\xi',x)) \), the \( \text{LWP} \) must continue with the tax adjustment policy by \( \tau(\xi',\xi',x) > \tau(\xi,\xi',x) \); however, adjusting now upon the welfare policy \( \xi' \) and proposing \( \xi'' > \xi' \), the new deficit becomes \( B(\xi'') > x \cdot \tau(\xi',\xi',x) \cdot W(\xi') \). These improvements \( u > u'' > u' \) initiate a sequence of poverty policies \((...\xi'' > \xi' > \xi,...)\) and after-tax LI residues \((...u > u'' > u',...)\). Thus, the conditions \( u = u'' \) and \( \xi = \xi'' \) can never be met, as this would contradict the assumption that the equation \( u = \pi(\xi, \tau(\xi,x)) \) cannot be resolved for \( \xi \). For this reason, the sequence \( ...,\xi'' > \xi',... \) is infinite. 

The chain of reasoning regarding \( u' > u \) is similar to above and is presented as a set of instructions. It should first be noted that, at low values \( u' > u'' > u \), even when taxes are low, there would always be a surplus to finance the LI benefits and subsidies. However, the surplus masks a contradiction, since it is clear that, at low values of the after-tax residue parameter \( u \), benefits financing can always be balanced.

<table>
<thead>
<tr>
<th>Replace</th>
<th>to implement an improved by to make a decline in</th>
</tr>
</thead>
<tbody>
<tr>
<td>– better off</td>
<td>– worse off</td>
</tr>
<tr>
<td>– improve</td>
<td>– decline</td>
</tr>
<tr>
<td>– improvement</td>
<td>– deterioration</td>
</tr>
<tr>
<td>– to claim for LI subsidies</td>
<td>– that LI subsidies have been revoked</td>
</tr>
<tr>
<td>– deficit</td>
<td>– surplus</td>
</tr>
<tr>
<td>– ≥,&gt;</td>
<td>– ≤,&lt;</td>
</tr>
</tbody>
</table>

Transpose: an increase with a decrease

In what follows, we investigate the targets \( \langle u, g \rangle \in S^g \) of the left- and right-wing politicians. The
consensus occurs at outcomes \( \phi, \xi \Rightarrow z, x, \alpha, \tau/(u, g) \) under the constraint that the variation of policy \( \xi \) does not improve the position of the left-wing politicians; rather, the policy emerges as the point on the contract curve \( S_b = u(g) \) as fiscally idempotent outcome.

For fiscally idempotent outcomes, the variables of after-tax residue \( u \), slice \( x \), policy \( \xi \), and tax rate \( \tau \) depend on each other. The slice \( x = x^o \), if settled as eventual agreement, redirects the residue \( u = \pi(\xi, \tau(\xi, x^o)) \) to become a function \( u = u(\xi, x^o) \). Thus, the peak policy \( u \) with regard to the best welfare policy can be expressed as

\[
\xi^o = \arg \max_{\xi} u(\xi, x^o)
\]

(A3.1)

**Lemma.** Let us assume that left-wing politicians do not shift from the slice \( x = x^o \) and that the volatility-constraint (4) solves for two different policies \( \xi_1 < \xi_2 \). Let the tax sacrifice \( t(\xi, x^o) = \tau(\xi, x^o) \cdot (\xi - \phi) \) be a differentiable function of \( \xi \) progressively increasing with \( \xi \) to increase within the closed interval \( [\xi_1, \xi_2] \)—namely, the derivatives

\[
\left. \frac{\partial}{\partial \xi} t(\xi, x^o) \right|_{\xi=\xi_1} > 0, \quad \left. \frac{\partial}{\partial \xi} t(\xi, x^o) \right|_{\xi=\xi_2} < 0 \quad \text{and} \quad \left. \frac{\partial^2}{\partial \xi^2} t(\xi, x^o) \right|_{\xi=\xi_2} > 0.
\]

In such situation, the poverty line residue \( u(\xi, x^o) = \xi - t(\xi, x^o) \) is a single \( \triangleright \)-peaked function of \( \xi \).

**Corollary.** There exists a unique interior policy \( \xi^o \) maximizing \( u \) at \( \frac{\partial}{\partial \xi} u(\xi, x^o) \bigg|_{\xi=\xi^o} = 0 \).

Provided that the conditions of the lemma are fulfilled, the discussion that follows concerns the necessary and sufficient conditions for the fiscally idempotent policy \( \xi \) to occur at the contract curve.

**Observation 2.** Let us assume that the volatility-constraint (4) is differentiable from its variables. The after-tax residue \( u = u(\xi, x^o) \) is differentiable and single peaked with respect to the policy \( \xi \) within some closed interval \( [\xi_1, \xi_2] \). For a fiscally idempotent outcome \( \phi, \xi^o \Rightarrow z^o, x^o, \alpha, \tau^o/(u^o, g^o) \) to occur on the contract curve \( S_b = u(g) \), it is necessary and sufficient that the policy \( \xi^o \) solves the set of equations:

(i) \[
\left. \frac{\partial}{\partial \xi} L(\xi, x^o, u^o) \right|_{\xi=\xi^o} = 0, \quad \text{where} \quad u^o = u(\xi^o, x^o) \quad \text{provided that}
\]

(ii) \[
\left. \frac{\partial}{\partial u} L(\xi^o, x^o, u) \right|_{u=u^o} \neq 0.
\]
Proof

Necessity. Let the fiscally idempotent outcome \( \phi, \xi^o \Rightarrow z^o, x^o, \alpha, \tau^o, \langle u^o, g^o \rangle \) on the contract curve \( S^o_b = u( g ) \) maximize (A3.1) at \( u^o = u( \xi^o, \tau( \xi^o, x^o ) ) \). Varying \( \xi \) in the vicinity of \( \xi^o \) of the outcome \( \phi, \xi^o \Rightarrow z^o, x^o, \alpha, \tau^o, \langle u^o, g^o \rangle \) and substituting \( u = u( \xi, \tau( \xi, x^o ) ) \) into the volatility-constraint (4), we obtain an identity \( L( \xi, x^o, \pi( \xi, \tau( \xi, x^o ) ) ) \equiv 0 \). Within the proximity of \( (\xi^o, u^o) \), the following equation holds for variables \( \xi, u \):

\[
\frac{\partial}{\partial \xi} L(\xi, x^o, u^o) + \frac{\partial}{\partial u} L(\xi, x^o, u) \cdot \frac{\partial}{\partial \xi} \pi(\xi, \tau(\xi, x^o)) = 0, \tag{A3.2}
\]

from which we deduce the necessity statement for \( \xi = \xi^o \) and \( u = u^o \).

Sufficiency. Suppose the condition (ii) holds. Let (i) solve for \( \xi^o \) at the fiscally idempotent outcome \( \phi, \xi^o \Rightarrow z^o, x^o, \alpha, \tau^o, \langle u^o, g^o \rangle \). Combining (i) and (A3.2), we conclude that

\[
\frac{\partial}{\partial \xi} \pi(\xi, \tau(\xi, x^o)) \bigg|_{\xi=\xi^o} = 0.
\]

The sufficiency clause (A3.1) holds, since \( u = u(\xi, x^o) \) is a strictly convex function of \( \xi \).

Proof of observation 3. The clause is correct, provided that there exists a fiscally idempotent policy \( \delta \) for the implementation of the pair \( \{d_1, d_2\} \). In order to identify such a policy, we first replace the variable \( g \) with \( d_2 \) in the expression for the constraint (1). Next, we extract the expression for \( \tau = \frac{B(\delta)}{W(\delta)} + d_2 \) from (1) and substitute it into \( (1- \tau) \) ... of the constraint (3), where \( u \) should be replaced by \( d_1 \) in advance. By simplifying, we arrive at the statement of the observation.

Sketch of the proof (observation 5). Looking at the wealth-tax value \( \tau > \tau_{\min} \), for any outcome \( ..., \tau, \langle u, g \rangle \in S^o_b \), one may indeed prefer a counter outcome as a motion \( ..., \tau, \langle u', g' \rangle \), which outlines \(..., \tau, \langle u' > u, g' < g \rangle \) or \(..., \tau, \langle u' < u, g' > g \rangle \). As the contract curve \( S^o_b = u( g ) \) is a curve of efficient preferences \( \langle u, g \rangle \) guaranteeing the poverty line residue \( u(g) \), someone could put a motion \( u' > u^o \) or \( g' > g^o \) against an outcome \( ..., \tau > \tau_{\min}, \langle u^o, g^o \rangle \). We argue that, in order to fulfill the emphases and requests of taxpayers', it is necessary to carry out political consent using the proposal \( ..., \tau_{\min} = \tau(\lambda), \langle u^o = u(\lambda), g^o = g(\lambda) \rangle \).
A4. Mathematical derivation

\[ \tau \cdot W(\xi) = B(\xi) + g \]

**Delivery-constraint:** the size of the welfare pie, i.e., the average tax returns amount equals the average monetary value per capita of primary goods plus average of non-primary goods \( g \).

\[ B(\xi) = x \cdot \tau \cdot W(\xi) \]

**Budget-constraint** imposed on the LI subsidies finance in accordance with the slice \( x \) of the wealth-pie—the tax revenue.

\[ u = (1 - \tau) \cdot (\xi - \phi) + \phi \]

**Stability constraint** that determines fiscally idempotent policy \( \xi \).

\[ u = \xi - \tau \cdot (\xi - \phi) \]

**After-tax residue constraint:** an alternative form of stability constraint, where \( u \) is after-tax position of a framed citizen with income \( \xi \). The left-wing politicians' targets.

Replacing \( \tau = \frac{B(\xi)}{x \cdot W(\xi)} \) from the budget-constraint into the stability constraint, we obtain the volatility-constraint (4) as stated:

\[ L(\xi, x, u) = (\xi - \phi) \cdot B(\xi) - x \cdot (\xi - u) \cdot W(\xi) = 0 \]

that amalgamates budget-constraint and after-tax residue. Contract curve (5) is thus given by:

\[ D(\xi, x, u) = L'_{\xi}(\xi, x, u) = \left[ (\xi - \phi) \cdot B(\xi) - x \cdot (\xi - u) \cdot W(\xi) \right] , = 0 ; \]
\[ L'_{\xi}(\xi, x, u) = B(\xi) + (\xi - \phi) \cdot B'(\xi) - x \cdot W(\xi) - x \cdot (\xi - u) \cdot W'(\xi) = 0 . \]

The last expression may be rewritten as:

\[ D(\xi, x, u) = B(\xi) + (\xi - \phi) \cdot B'(\xi) - x \cdot [W(\xi) + (\xi - u) \cdot W'(\xi)] = 0 . \]

Extracting \( x = \frac{(\xi - \phi) \cdot B(\xi)}{(\xi - u) \cdot W(\xi)} \) from the volatility-constraint (4), we can substitute variable \( x \) into the rewritten expression for \( D(\xi, x, u) \). The substitution results in the following expressions:

\[ B(\xi) + (\xi - \phi) \cdot B'(\xi) - \frac{(\xi - \phi) \cdot B(\xi)}{(\xi - u) \cdot W(\xi)} \cdot [W(\xi) + (\xi - u) \cdot W'(\xi)] = 0 , \text{ or} \]
\[ \left[ B(\xi) + (\xi - \phi) \cdot B'(\xi) \right] \cdot \frac{(\xi - u) \cdot W(\xi) - (\xi - \phi) \cdot B(\xi) \cdot [W(\xi) + (\xi - u) \cdot W'(\xi)]}{(\xi - u) \cdot W(\xi)} = 0 . \]
Provided that \((\xi - u) > 0\) and \(W(\xi) > 0\), we can conclude that the following is true:

\[
[B(\xi) + (\xi - \phi) \cdot B'(\xi)] \cdot (\xi - u) \cdot W(\xi) - (\xi - \phi) \cdot B(\xi) \cdot [W(\xi) + (\xi - u) \cdot W'(\xi)] = 0.
\]

This allows writing the sub-expression \((\xi - u)\) in the form:

\[
\{ [B(\xi) + (\xi - \phi) \cdot B'(\xi)] \cdot W(\xi) - (\xi - \phi) \cdot B(\xi) \cdot W'(\xi) \} \cdot (\xi - u) - (\xi - \phi) \cdot B(\xi) \cdot W(\xi) = 0.
\]

As a consequence of presenting the sub-expression \((\xi - u)\) in the form given above:

\[
\begin{align*}
\xi - u &= \frac{(\xi - \phi) \cdot B(\xi) \cdot W(\xi)}{B(\xi) + (\xi - \phi) \cdot B'(\xi)} \cdot W(\xi) - (\xi - \phi) \cdot B(\xi) \cdot W'(\xi). \\
u &= \xi - \frac{(\xi - \phi) \cdot B(\xi) \cdot W(\xi)}{B(\xi) + (\xi - \phi) \cdot B'(\xi)} \cdot W(\xi) - (\xi - \phi) \cdot B(\xi) \cdot W'(\xi). 
\end{align*}
\]

We can now substitute the tax rate \(\tau\) from the delivery-constraint into the after-tax residue constraint. The result will be \(u = \xi - \frac{B(\xi) + g}{W(\xi)} \cdot (\xi - \phi)\). After replacing the result into the observed \(u\) expression, we obtain:

\[
\begin{align*}
\xi - \frac{B(\xi) + g}{W(\xi)} \cdot (\xi - \phi) &= \xi - \frac{(\xi - \phi) \cdot B(\xi) \cdot W(\xi)}{B(\xi) + (\xi - \phi) \cdot B'(\xi)} \cdot W(\xi) - (\xi - \phi) \cdot B(\xi) \cdot W'(\xi); \\
\frac{B(\xi) + g}{W(\xi)} \cdot (\xi - \phi) &= \frac{(\xi - \phi) \cdot B(\xi) \cdot W(\xi)}{B(\xi) + (\xi - \phi) \cdot B'(\xi)} \cdot W(\xi) - (\xi - \phi) \cdot B(\xi) \cdot W'(\xi); \\
[B(\xi) + g] \cdot (\xi - \phi) &= \frac{(\xi - \phi) \cdot B(\xi) \cdot W(\xi)}{B(\xi) + (\xi - \phi) \cdot B'(\xi)} \cdot W(\xi) - (\xi - \phi) \cdot B(\xi) \cdot W'(\xi); \\
B(\xi) + g &= \frac{B(\xi) \cdot W(\xi) \cdot W(\xi)}{B(\xi) + (\xi - \phi) \cdot B'(\xi)} \cdot W(\xi) - (\xi - \phi) \cdot B(\xi) \cdot W'(\xi); \\
g &= \frac{B(\xi) \cdot W(\xi) \cdot W(\xi)}{B(\xi) + (\xi - \phi) \cdot B'(\xi)} \cdot W(\xi) - (\xi - \phi) \cdot B(\xi) \cdot W'(\xi) - B(\xi). 
\end{align*}
\]
We can now impose the denominator in the last expression for $g$ on sub-expression for $(\xi - \phi)$, which can be written as:

$$[B(\xi) + (\xi - \phi) \cdot B'(\xi)] \cdot W(\xi) - (\xi - \phi) \cdot B(\xi) \cdot W'(\xi) =$$

$$= B(\xi) \cdot W(\xi) + (\xi - \phi) \cdot [B'(\xi) \cdot W(\xi) - B(\xi) \cdot W'(\xi)]$$

Continuing with the expression for $g(\xi)$, we can replace the denominator transformed above:

$$g = \frac{B(\xi) \cdot W(\xi) \cdot W'(\xi)}{B(\xi) \cdot W(\xi) + (\xi - \phi) \cdot [B'(\xi) \cdot W(\xi) - B(\xi) \cdot W'(\xi)]} - B(\xi);$$

$$g = \frac{B(\xi) \cdot W(\xi) - B(\xi) \cdot [B(\xi) \cdot W(\xi) + (\xi - \phi) \cdot [B'(\xi) \cdot W(\xi) - B(\xi) \cdot W'(\xi)]]}{B(\xi) \cdot W(\xi) + (\xi - \phi) \cdot [B'(\xi) \cdot W(\xi) - B(\xi) \cdot W'(\xi)]};$$

Now, both the nominator and the dominator can be divided by $B(\xi) \cdot W(\xi)$, yielding:

$$g = \frac{W(\xi) - B(\xi) \cdot \left\{ B(\xi) \cdot W(\xi) + (\xi - \phi) \cdot [B'(\xi) \cdot W(\xi) - B(\xi) \cdot W'(\xi)] \right\}}{B(\xi) \cdot W(\xi) + (\xi - \phi) \cdot [B'(\xi) \cdot W(\xi) - B(\xi) \cdot W'(\xi)]}.\right\}$$

Let us define $v(\xi) = 1 + (\xi - \phi) \cdot \left( \frac{B'(\xi)}{B(\xi)} - \frac{W'(\xi)}{W(\xi)} \right)$, as this allows us to evaluate the expression for the right-wing politicians' targets on public but vital goods as:

$$g(\xi) = \frac{W(\xi) - B(\xi) \cdot v(\xi)}{v(\xi)} = \frac{W(\xi)}{v(\xi)} - B(\xi).$$

In accordance with the delivery-constraint, the size of the wealth pie $\tau(\xi) \cdot W(\xi)$ equals $B(\xi) + g(\xi)$. Consequently, the tax rate is given by:

$$\tau(\xi) = \frac{B(\xi) + g(\xi)}{W(\xi)} = \frac{B(\xi) + \left( \frac{W(\xi)}{v(\xi)} - B(\xi) \right)}{W(\xi)} = \frac{1}{v(\xi)}.$$

Replacing the $\tau(\xi) = \frac{1}{v(\xi)}$ in the after tax residue $u = \xi - \tau \cdot (\xi - \phi)$, we can finally evaluate the expression for the left-wing politicians' targets on basic goods as:

$$u(\xi) = \xi - \frac{(\xi - \phi)}{v(\xi)}.$$