“The hardest thing in the world to understand is the income tax”

Albert Einstein

Article

The Left-Right Wing Political Power Design:
Welfare Policy Dilemma–Subsidizing Citizens' Low-Incomes

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Abstract: This paper examined the controversy of whether welfare policy should be just and fair to all citizens. Based on the assumptions similar to those implicit in Negative Income Tax, the resolution of the controversy did not favor increasing the social security grants and setting welfare benefits at overly high levels. In order to investigate this issue, two key actors/politicians and an implicit partaker took part in negotiations for the well-being of citizens. While both politicians campaigned for public goods and services, their expectations were interpersonally incomparable. The 1st actor, representing left-wing politicians, struggled for citizens' legal and moral rights to primary needs and for the delivery of basic goods. On the other hand, the 2nd actor represented right-wing politicians and advocated for citizens' needs for the delivery of non-primary but vital public goods and services. Finally, the 3rd actor, the implicit partaker, embodied the majority of citizens. The majority preferred personal consumption to moral and social understanding. The aim of society as a whole was to ensure by left-right wing political power design the just and fair public spending to all citizens.

Keywords: bargaining; welfare policy; public goods; taxation; voting

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1. Introduction

As the welfare policy of the state presupposes the existence of both a functioning market economy and a democratic political system, its hallmark is that the distribution of public goods and services is governmental responsibility and obligation. The term public in this context refers solely to wealth redistribution. In particular, an obligation to ensure that those on low-incomes are awarded appropriate levels of social benefits and subsidies ensures a more egalitarian allocation of wealth than can be provided by the free market. In this scenario, politicians face a dilemma of whether such allocation is just and fair to all citizens. The solution depends on many factors, including the characteristics and views of the main benefactors of wealth redistribution. In the absence of a universal definition, we use the term wealth narrowly defined as "prosperity or a commodity" $W$, delivered through tax channels, and distributed by the State. The average taxable income per capita represents the wealth.

Political competition, related to wealth redistribution, often fosters debate regarding what the state "should" or "should not" deliver. Wider and more substantial welfare benefits and subsidies could be problematic, as they might encourage certain behaviors, such as low savings or productivity when economic security is guaranteed. Similarly, they may lead to high wage demands, in particular, as an incentive to remain in employment when unemployment benefits are substantial and are compensated by high tax rates $\tau$. While high taxes also become an incentive for entering a black labor market that avoids paying taxes, or moonlighting, i.e., holding multiple jobs—the high benefits typically undermine social and geographical mobility. Finally, few would opt for working and studying because it simply does not pay them to do so. In sum, excessive benefits might result in the human capital not developing quickly and well enough, c.f., "...implicit support to those waiting on benefits looking for the 'right type of job' or a job that pays well enough," Oakley & Saunders [1], 2011, p. 20.

The primary goal of this study is to express the view that arguments often advocating in favor of higher benefits and subsidies are false. Beyond the negative perception of higher benefits, it is also reasonable to believe that distribution of citizens' incomes $\sigma$ is, perhaps, the only target for control and an exclusive source of information for assessing the amount of benefits available. Due to the absence of panel data, based on credible incomes' distribution, and experimental support and justification for our view, we are approaching this issue from a more theoretical perspective. However, despite this key weakness, the solution of the welfare policy dilemma, based on numerical simulations, yields benefits sufficiently close to be considered a realistic match (see Table 1) to "what amounts to a moving poverty line at 50% of median income," c.f., Fuchs [2] point; Bowman [3], 1973, p. 55. In support of this approach, it is worth noting that Rawls [4], in 1971/2005, on p. 98 pronounced the Fuchs point as an alternative to the measurement of poverty with no reference to social position. The motive of the current study is thus to point—while acknowledging that a few examples clearly cannot make a trend—that we presented a theoretical confirmation for the claim recognizing the poverty line, defined as 50% of the median income, as a realistic political consensus.
Political consensus not always take place in reality. We try, therefore, to design a political platform for welfare policy that ought to be implemented. This approach does not require analysis neither voting system nor a scheme by which voters-citizens express their arguments. In doing so, however, we analyze political power indicators as replications \((\alpha, 1 - \alpha)\), \(0 < \alpha < 1\), in Kalai’s bargaining game [5], 1977, on 1$ division. For a power \(\alpha\) is appropriate to adopt the ability to negotiate, or be in the position to request financial support in greater extent than the opposite site, etc. Similar interpretation of players’ power dynamic may be found in Mullat [6], 2014. In short, we adopted the view of Roberts, [7], 1977, p. 329, who noted, “The point is not whether choices in the public domain are made through a voting mechanism but whether choice procedures mirror some voting mechanism.”

In our scheme, citizens earning low-incomes (below a certain level—the poverty line \(\xi\)) receive subsidies, whereas those with higher incomes—above the aforementioned level—do not. In this regard, it should be noted that, in 1962, Milton Friedman [8], 2002, pp. 190-195, proposed a similar scheme of wealth redistribution, combined with flat tax, called the negative income tax—the NIT. According to the rules and norms of the NIT, low-income citizens receive a subsidy proportional to the difference between their earnings and the predetermined NIT poverty line. Most importantly, the total—the sum of the key income and the NIT subsidy—is not subject to taxation. When levying taxes in compliance with the tax rules and norms in force for all, inclusive of low-income citizens, the arithmetic of taxes would have the same result. Although the total income of low-income citizens is now taxable, they would still be eligible for subsidies in the NIT spirit, similar to the widely adopted LI—low-income—subsidies. The known drawback of such an approach, and the subsidies, in particular, stems from the issue of social abuse among low-income citizens. In order to mitigate its effects, we introduce the so-called hazard of working incentives, referred to as the h-effect.

We consider a masquerade of life or scenario of realistic utopia—a theoretical model of visionary politicians. In this scenario, two actors/politicians, akin to two political coalitions, are playing a bargaining game, each trying to implement its own wealth redistribution policy. Left-wing politicians tend to see the disproportion in private consumption, unjust wealth redistribution, profit motive, and private property as the main sources of socioeconomic evil. Right-wing politicians, on the other hand, tend to focus on regulating business and financial risks, encouraging the government’s use of its powers in combating corruption, criminal violence and commercial fraud. While left-wing politicians prefer an equitable sharing of the available stock of goods and services here and now, both sides are aware of the citizens’ sacrifices—in terms of direct contribution of a part of their income to the funding of welfare benefits and public goods. However, applying the rules and norms of wealth redistribution pertaining to the reliance on the elevated subsidies would increase the quantity of the subsidies to be delivered. Consequently, citizens will have to meet a greater tax burden. This is not ideal, given that the tax burden and private consumption always lie at the heart of citizens’ economic and political wants. These wants lead to controversy of interests of voters’ majority, who hold power in electing political parties. As a result, they are instrumental in the competition between two key representatives—left-right wing politicians—and their views on tax policies.
We hope that these brief remarks have clarified some goals of the state, allowing us to conclude that welfare policy in a representative democracy always faces controversial interests of politicians and citizens. The aim of this study is thus also to shed light on how a political consensus is reached and whether it reflects a criterion of tax policy that ensures the least burden. To address this issue, as already stated, we focus our analysis on two visionary politicians. For the purpose of the following discussion, we granted these politicians a political mandate to initiate proposals ensuring that subsidies are allocated to citizens that are in need. We will assume that, expenses shall be constrained on subsidies in balancing the books accounting for finance public goods and services. This ensures that negotiations are under the control of the citizens, forcing the politicians to act within the imposed budget-constraint in order to pledge safe funding for their proposals. While trying to reduce the after-tax income inequality, the politicians in the roles of left-right actors are committed to ensuring that the wealth is redistributed fairly.

It is now high time to clear up some of the assumptions/limitations in the analysis of a hypothetical behavior of citizens appointed for three distinct roles in the negotiations—as those of left-right wing politicians and voters-citizens. Throughout the entire study, we stressed the incomparability between the wants of the left-wing politicians struggling to ensure adequate access to basic goods, and the right-wing politicians advocating for non-primary but vital goods and services. In the analysis, we implicitly assumed that politicians did not have adequate knowledge about peoples' needs in more primitive environment at their disposal and could only work with the monetary payoffs specification. Politicians could not know that the provision of equivalently valued public services was not a perfect substitute. For example, we assumed that politicians did not have any information on how the income of households was assembled and used to buy private health insurance or services of nursing housing, etc. Thus, we did not merit the debate on what was right or wrong in the economic or political environment involving left-right wing politicians and voters-citizens. In short, we were not interested in a democratic context of voters' prototypes/characteristics. Despite significance of prototypes, we simplified voters' behavior to binary voters emanating a risk \( q > 0 \) of premature breakdown of negotiations voting either for left- or right wing politicians. Therefore, we referred to the tax-revenue in accord with voters' preferences as the "wealth-pie" \( \tau \cdot W \) being divided into two shares \((x, y)\): \( x \) of various social benefits, and \( y \) of public goods, \( x + y = 1 \). Enrichment of voters' characteristics would spoil beyond doubt the delicate balance between the motives of our study and the theoretical framework, which already is technically sophisticated.

Before delving deeper into the analysis, in Section 2, we specify the category of payoffs functions \( u(\xi, x), g(\xi, y) \) and taxes \( \tau(\sigma, x) \) required for the model validity. The Section 3 walks through the skeleton of the article's main body given in Sections 4-6. In Section 4, we disclose in details fiscally safe welfare policy in amalgamation with imposed budget-constraint for financing subsidies. Referred to as volatility-constraint, the amalgamation dynamically restricts the h-effect—an inverse working incentives phenomenon of low-income citizens. In Section 5, citizens' ambivalence and multifaceted welfare policy perception are discussed from the alternating-offers' game perspective. Here, the policy on poverty associates the left-right wing politicians payoffs with functions \( u(\xi, x) \) and \( g(\xi, y) \). Given the incomes \( \sigma \) density distribution \( P(\sigma, \xi) \) it would be possible to obtain an analytical solution to the game. The calculus of indicators \( (\alpha, 1-\alpha) \) fulfilled the political power design in Section 6. The discussion of the results follows in Section 7, while concluding remarks are presented in Section 8.
Roadmap. Because of the narrative complexity, the reader, apparently, will be faced with the inability to trace the material of the paper in one chain, moving from previous to the following section. Therefore, to ensure reading fluency, the Section 3 will help the reader to identify the problems most relevant, in particular, the Pre-equity condition of breakdown of the negotiations. We recommend, first to learn the material of Section 3.1, and only then, if it is desirable, to move along Section 4. When mastered Section 3.2, it will be useful to succeed with Section 5, and finally, when mastered Section 3.4 complete the motion along political power design in Section 6. The reader who even does not wish to go into detailed technical content may well drop sections 2,4-6 continuing reading from section 7 and still get an idea of our work. With respect to Section 3.3, our scheme here involves the pre-equity of breakdown of the negotiations and, in contrast, does not require further clarification.

2. Preliminaries

In current study, an income $\sigma$ equal to the poverty line $\xi$, $\xi \in [\xi_1, \xi_2]$, parameterizes all arguments and functions. In this vein, we adopt quantitative measurement, whereby we utilize a scale quantum as an average income over income $\sigma$ density $P(\sigma, \xi)$ distribution, $0 \leq \sigma < \infty$. The average establishes the ratio scale. Doing so, we suggest $u(\xi, x) = (1 - \tau(\xi, x)) \cdot (\xi - \phi) + \phi$ — the after-tax residue of income $\sigma = \xi$ to signify the 1st actor's social position at the specified scale—the left-wing political wants. We applied the residue formula based on Malcomsons' [9] model, 1986, p. 266 with a personal allowance parameter $\phi$, $0 < \phi < \xi$, determined by the tax bracket $[\phi, \infty)$. The 2nd actor—the right-wing political objective $g(\xi, y)$ is the non-basic goods per capita. A citizen $\sigma = \xi$ is referred to as marginal citizen. The 3rd actor—the implicit partaker—embodies all voters behind the scene—some voting in hope for politicians will minimize their tax obligation $\tau(\sigma, x)$, or some voting for the other expectations of welfare policy. This is typical public finance dilemma of efficient division $(x, y)$ of the tax-revenue into shares $x + y = 1$. The dilemma is set up by the alternating-offers' bargaining game $\Gamma(q)$ with premature risk $q$, $0 < q << 1$, of negotiations breakdown. This game is thoroughly described in the literature—a variant of Osborn and Rubinstein [10], 1990, p. 31. We do not go beyond [10], so there is no need for a detailed description of the game. The solution of this game converges when $q \rightarrow 0$ into Nash axiomatic approach [11], 1950. This relationship between an axiomatic and the one that suggests the alternating-offers' bargaining solution is well known from [10], 1990, p. 75.

When negotiating on finance issues, under the guise of a "wealth-pie workshop" the politicians will allegedly try to divide the wealth-pie in a rational and efficient manner. In doing so, the politicians will meet the tax $\tau(\sigma, x)$ increase—so does the wealth-pie, when increasing the poverty line $\xi$. Decrease of taxes yields the reverse effect. However, while taxes vary, the division will depend upon the characteristics and expectations of the bargainers performing it. Indeed, the left-right wing political wants $u(\xi, x) \cdot \text{basic and objective } g(\xi, y)$ of non-basic goods have been marked as being-controversial. We illustrate this tax controversy by elevated single-peaked frontier of $u(\xi, x)$, the $\frac{53}{52}$-share in Figure 1, which corresponds to the lower, but progressively increasing, concave frontier of $g(\xi, y)$, the $\frac{53}{52} -$share in Figure 2, as well as for another division of the pie, into shares $(x = \frac{53}{52}, y = \frac{53}{52})$. We believe, that, while $(x = \frac{53}{52}, y = \frac{53}{52})$ highlights the left-wing political wants, the share $(\frac{53}{52}, \frac{53}{52})$ highlights those of the right-wing political objective. This premise appears to be crucial for understanding our primary goal in resolving the welfare policy dilemma.
In support of the aforementioned belief, the political payoffs in general, as shown in Figures 1-2, emerge within a two-man economy endowed by citizens' income abilities marginalized at the level of poverty line. According to Black [12], 1948, p. 27, single peakedness plays a major role in collective decision-making when the decision is arrived at by vote. The payoffs of two actors, shaped in this way, are called non-conforming—incomparable—impossible to match through a monotone transformation, Narens & Luce [13], 1983. The single peakedness turns up in accord with Malcomsons' tax residue formula $u(\xi, x)$ when the terms of contract commit the actors to shares $(x, y)$ provided the expenses covered by flat taxes will balance the books accounting for subsidies, Figure 1. The increase of poverty line requires an excessive increase of taxes, which in turn opens a coincident increase of non-basic goods $g(\xi, y)$, Figure 2. An opposite scenario of increasing non-basic goods $g(\xi, y)$ equally requires an excessive tax increase, which opens up the possibility of increasing poverty line.

Following the traditional procedure of how to share the pie in the alternating-offers' game, when the pie is desirable at all the times, the politicians (bargainers)—changing roles—commit to shares $(x, y)$, $x + y = 1$. According to the shares $(x, y)$, the valid rules and norms of wealth redistribution, which guarantee a desirable level of subsidies, require establishing a poverty line $\xi$ parameter. However, an efficient division of the wealth-pie—as a result of single-peaked $\sim$-curves in Figure 1—no longer represents any traditional bargaining procedure as, instead of division, the procedure can be resettled. Indeed, we can proceed at distinct levels of one parameter—inside the poverty line interval $[\xi_1, \xi_2]$—reflecting the scope of negotiations. In fact, in 2007, Cardona and Ponsatti [14], p. 628, also noted that "the bargaining problem is not radically different from negotiations to split a private surplus," when all the parties in the bargaining process have the same, conforming expectations. This argument applies even when the expectations of the first player are principally non-conforming, i.e., single-peaked, rather than concave. In our study of non-conforming expectations, the scope of negotiations on the "contract curve" allows for omitting the axiom of "Pareto efficiency," known since Roth [15], 1977, as "well defined bargaining problem." The well-defined problem $(x, y)$ of the wealth-pie division, as a substitute, can now be solved (resettled) inside the poverty lines' interval $[\xi_1, \xi_2]$. 
2.1. Settings

In accordance with Friedman's NIT system, we assumed that, for the unfair subsistence of the less fortunate citizen $\sigma < \xi$, the amount $r \cdot (\xi - \sigma)$, $0 < r \leq 1$, of subsidizes, serves as a monetary compensation designated for purchasing an eligible "poverty basket" of food, clothing and shelter, fuel and lights, etc. According to Rawls, [4], 1971/2005, p. 92, "primary goods are things which it is supposed a rational man wants whatever he wants." In defining the parameter $\xi$ in this manner, it becomes contingent on financing subsidies. This can be achieved by assuming that higher values of the poverty line $\xi$ require an increased marginal tax rate $\tau(\sigma, x)$. Therefore, while increasing the wealth-pie through tax channels, we assumed a positive acceleration $\tau''(\sigma, x) > 0$ of the tax rate $\tau(\sigma, x)$; $\tau'(\sigma, x) > 0$ for any of citizens, inclusive all of those who indicate the marginal income $\xi$ of a citizen denoted by $\sigma = \xi$.

As we already revealed, we defined per capita, upon the marginal citizen $\sigma = \xi$, the left-wing political wants using tax residue $u(\xi, x)$, and the right-wing political objective $g(\xi, x)$, referred to as "public or non-basic goods." With the proviso that politicians commit to the shares $(x, y)$, we concluded that $u(\xi, x)$ was a single $\cap$-peaked due to the tax rate $\tau(\xi, x)$ increase upon $\xi$. While objective $g(\xi, x)$ of right-wing politicians decrease with $x$ increase, they increase with elevating $\xi$ for the reasons of $\tau(\xi, x)$ positive acceleration. Payoffs $(u, g)$ are considered analytic functions $u(\xi, x)$, $g(\xi, x)$. Given the interval $[\xi_1 \leq \xi \leq \xi_2]$, referred to as the scope of negotiations, $u(\xi, x)$ reflects single $\cap$-peakedness—$u'' < 0$ upon $\xi$ increase, $u'_x(\xi_1, x) > 0$, $u'_x(\xi_2, x) < 0$. Upon an increase in $x$, the payoffs $u(\xi, x)$ become convex, $u'' > 0$, $u'_x > 0$, whereas following the increase in $\xi$, payoffs $g(\xi, x)$ should be concave with $g'_x > 0$, $g''_x > 0$. With increasing $x$, payoffs $g$ always decrease; in other words, in both circumstances, either $g''_x > 0$ is convex, or $g''_x < 0$ is concave.

3. Relevant trends and issues

In the extant literature, the welfare policy issues are usually addressed separately. In our view, a much deeper analysis is obtained addressing them together. In particular, our wealth-pie workshop concept, adopting jointly four issues—(a) public finance, (b) alternating-offers' game, (c) negotiations' collapse analysis, and d) political power design—leads to a more narrative bargaining procedure.

To explain the root cause of the results and find such bilaterally acceptable solutions to the game, we will try to visit all of the classrooms in our workshop, in order to bring the economic and political content to the surface in a rigorous analytical form. Our goal is to lay the foundation for a more constructive welfare policy comprehending the meaning of following four narratives:

| Fiscal policy | During the delivery to its final destinations, provided that the books accounting for the subsidies finance have been balanced a priori, the wealth-pie must remain balanced throughout and in spite of volatility in the economy; |
| Negotiations | The left-right political bargaining on how to share the wealth-pie complies with the rules and norms of the alternating-offers' bargaining game; |
| Pre-equity of breakdown | Breakdown, or threat, point directly affects the bargaining solution. Pre-equity guarantees equal conditions for players before the bargaining game starts; |
| Political power design | Bringing a motion to a vote is necessary to address the majority opposition to high taxes and excessive public spending. Whether it is viewed as positive or negative, or whether it ought to be acknowledged or not, rejected or accepted, this motion must be politically designed in advance. |
In our wealth-pie workshop, these four narratives can be understood as obligations/constrains to be met by welfare policy rules and norms, c.f. "Rational man" deliberation, Rubinstein [16], 1998, p. 7. This interpretation will enable us to affirm the view under which the narratives are embedded into the welfare policy of the state. In evaluating the welfare policy from this perspective allows us to demonstrate that the analysis can be subject to and performed by computer simulations, see Appendix A2. Our initiative could also serve to unify the theoretical structure of economic analysis of public spending, evaluate the political power design of left-right wing politicians, or conduct systematic inquiry into impacts of governmental decisions and actions on wealth redistribution.

As the state has the duty to help the less fortunate, our study approaches wealth redistribution in a two-fold manner. First, it addresses the provision of basic necessities or goods, such as shelter and lights, clean and fresh water, nutrition,… before focusing on non-basic goods, including national defense, public safety and order, roads and highway systems,… Welfare policy economists, c.f., Reder [17], 1947, believe that just and fair wealth redistribution is achieved through the efficient division of society’s resources. When designing welfare policy, based on incomes $\sigma$ density distribution $P(\sigma, \xi)$, while trying to redistribute the wealth fairly, we must identify an effective approach to the division $(x^*, y^*)$ into shares $x^* + y^* = 1$ of basic $x^*$ and non-basic goods $y^*$. Fundamentally, the efficient shares $(x^*, y^*)$ of wealth-pie aims at just and fair delivery of all aforementioned goods, preserved traditionally as public goods. In our study, we refer to public goods as non-basic but vital goods, in contrast to, treating basic goods as fundamental. Incidentally, during the delivery of basic and non-basic goods to their end destinations, we treat both as public goods.

Suppose that, within the poverty interval $[\xi_1, \xi_2]$, the left-wing politicians have the necessary political power—when an offer is made, irrespective of its legitimacy—to control the poverty line $\xi$ independently. Given the single-peaked wants of the left-wing, in contrast to objective of their right-wing counterparts, the power the left-wing political enjoy is supposed to be strong enough to reach the peak of their expectations. In making these suppositions, we agree with Rawls' [4] statement, 1971/2005, p. 304, about the precepts of perfect justice: "The sum of transfers and benefits [...] from essential public goods should be arranged so as to enhance the emphases of the least favored consistent with the required saving and the maintenance of equal liberties."

An efficient usage of public resources implies that a consensus between left-right wing politicians might be reached. Despite various views to the contrary, we posit that the bargaining aimed at finding a just and fair division of basic vs. non-basic goods is an acceptable path to the bargaining dynamics. In particular, we believe that beyond some peak position the greater demands on basic goods would lead to the excessive decline in the quality of welfare services, as well as cause deterioration in access to all basic goods for all citizens. Based on this premise, we review relevant publications on economic and political behavior, which deal with the sociological and political wants of welfare using public finance. This is likely be the best starting point for visiting our wealth-pie workshop.
3.1. Fiscal policy, to be continued in Section 4

Public finance focuses on the revenue side of tax policy. In particular, it pertains to the budget formation, c.f., Formby and Medema, [18], 1995, aiming to provide a guaranteed level of welfare to citizens endowed by poor productivity. While the welfare policy is a separate matter, it is worth considering on the grounds of legal and moral rights of citizens. Empirical evidence consistent with legal obligations can be found in the literature on social policy. As noted by Saunders [19], 1997, “...poverty line. The line was initially set (in 1966) equal to the level of the minimum wage plus family benefits for one-earner couple with two children.” Similarly, a hypothesis consistent with moral obligations can be found in the literature of economic politics, Eichenberger & Oberholzer-Gee [20], 1996, Feld & Frey [21], 2002.

Musgrave, [22], 1959, examined two basic approaches to taxation—the "benefit approach" and "ability-to-pay," which put taxation into efficiency and equity context respectively. We intend to augment the existing standard of welfare policy by benefit approach, whereby we allocate a guaranteed amount of income for minimum taxes. We posit that the flat tax system, based on injecting optimal equity, according to the ability-to-pay principle of "proportional sacrifice," ensures that taxes remain fairly levied.

Taxation is a principal funding source of social costs and benefits. Thus, the first postulation in our welfare policy workshop (see above) discloses an obvious paradigm in social policy. According to the ability-to-pay principle, commonly adopted in public finance, in order to make the distortion of tax policies stable, the known terms of warranty must rely on exogenous taxes enforced on the productivity of citizens. The concept, proposed in 1996 by Berliant and Page Jr., [23], is a variant of the classic public finance and similar approaches, applicable when an agent characterized by a specific level of productivity does not shift his/her labor supply after all adjustments to the tax formula have been implemented. In other words, optimal taxation enforces optimal labor supply.

Yet another "treatment of policies," closely related to societal instability, entails equity of pre- and post-tax positions of citizens. Such a view, that demarcates between citizens, attracts the attention of economists and tax policy makers. In the view of Kesselman and Garfinkel, [24], 1978, credit tax-scheme analysis opposes the income-tested program in the rich-and-the-poor—two-man economy. Poverty measurements have also been addressed in the works of Sen, [25], 1976, Atkinson, [26], 1987, Ebert, [27], 2002, and Hunter et al., [28], 2002. Horizontal inequalities seem to occupy a place in Stewart's paper, [29], 2000, where the author reviewed the connection between wealth redistribution and economic growth. In a later study, Peñalosa and Wen, [30], 2004, investigated wealth redistribution, treating it as a form of social insurance. According to Tarp et al., [31], 2002, p. 8, "The poverty line acts as a threshold with households falling below the poverty line considered poor and those above poverty line considered nonpoor."

While we continue to rely on stabilization policy, we refer to welfare policy as idempotent, in order to highlight the policy particular type of the dynamics stability. For clarity, a choice operation (or decision) applied multiple times is deemed idempotent if, beyond the initial application, it yields the
same result, c.f., Malishevski [32], 1998, p. 422. Based on this definition, an idempotent scheme guarantees for politicians the ability to keep the pledges made during the election campaign as, once the decision is taken, it eliminates the need for further stabilization. In the attempt to assess and control the circulation of wealth through social and public organizations, we argue that, unless dynamic stabilization is not a required condition whenjustifying political decisions through public spending, it will be difficult to explain how the benefits and subsidies can reach all members of the society.

3.2. Negotiations, to be continued in Section 5

Bargaining is the key element of economics and the core of politics. However, “The interface between economics and politics is still in a primitive state in our theories but its development is essential if we are to implement policies consistent with intentions,” as pointed out by North, [33], 2005, p. 29. Feldstein, [34], 2008, p. 132, also noted: “Unfortunately, there is no reason to be pleased about the analysis in policy discussions of the efficiency effects...of the welfare consequences of proposed tax changes.” Similarly, Richter, [35], 2005, p. 387, in a review on “Handbook of New Institutional Economics” stressed, “...that the sociological analysis...and large institutional structures in economic life is still at an early stage...game theory, and computer simulation could help to further develop the new institutional approach...game theory might be a defendable heuristic device of NIE.”

It is realistic to imagine a scenario in which our actors play the “bargaining drama” of economic and political issues in our wealth-pie workshop. While visiting the workshop, the circulation of wealth is supposed to be dynamically stable. Under these conditions, the left-right wing politicians would be trying to implement their vision of the state welfare institutions: “These flimsy structures, however, are used by individuals to allocate resource flows to participants according to rules that have been devised in tough constitutional and collective-choice bargaining situations over time,” Ostrom, [36], 2005, p. 823. Bargaining has been a theme of a wide range of publications, including Alvin E. Roth, [37], 1985. However, it can be risky to bargain for both actors because the voters may defect to the other side if their terms are not met. This binary position of the voters fits particularly well into bargaining game with exogenous risk $q$, 0 < $q$ < 1, of breakdown, [10], 1990, p. 71.

We will first clarify political power asymmetric dynamics of both—the left-wing politicians, henceforth referred to as $LWP$, the 1st actor, benefiting from a power $\alpha$, 0 < $\alpha$ < 1, and the right-wing politicians $RWP$, the 2nd actor benefiting from a power 1 − $\alpha$. Indeed, numerous factors—such as economic growth, decline or stagnation, demographic shift or pit, political change or change in scarcity of resources, skills and education of the labor force, etc.—might often create fiscal imbalance in a desirable welfare policy due to the transfers of LI benefits and subsidies. Consequently, the size of the wealth-pie might be too small (i.e., not worth the effort), or too large (introducing mutual traps) to achieve a stabilized public spending mechanism. Thus, the actors may decide not to share the pie at all. To address this situation, as previously underlined, we assume that politicians participate in relevant public institutions. For instance, if the institutions does not meet their goal, $RWP$ are sufficiently legitimate to promote their own understanding of wealth redistribution to deliver the wealth "properly." For example, $RWP$ can enforce vital decisions by resource mobilization, retaliate for
breaches and criminal fraud, recruiting and managing volunteers and movements, solicit private contributions, etc. In other words, as Kalai, [5], 1977, p. 131, put it, RWP would rely on an "enthusiastic supporter." On the other hand, as LWP face a decay in political legitimacy for perfect justice, they cannot fully control RWP's actions and intentions when their political interests in the final agreement are incomparable. In these circumstances, RWP knowing its abilities and access to information might resort to agreeing with, or at least not resist, LWP's privileges to make arrangements upon the size of the pie. Hence, from the RWP's critical point of view, whether acting politically in common interest or not, it might be prudent to acknowledge LWP's welfare activities. Our insight is forward-looking in the sense that it aims to spot an alternative offers game solution, whereby both actors accept at once the proposals (moves) made by the other side. This clarifies the asymmetric dynamics of political power division between the LWP and RWP.

With these remarks in mind, any procedure of negotiating on shares, accompanied by poverty lines \( \xi \in [\xi_1, \xi_2] \), can be perceived as two sides of the same bargain's portfolio. It is irrelevant whether the players are bargaining on shares \((x, y)\) or trying to agree on poverty line. This highlights the main advantage of the parametric procedure—it brings about a number of different patterns of outcomes interpretations in the game, such as linking an outcome to the lowest tax rate, which is the most desirable sacrifice of voters' majority. In consideration of alternative ways—which describe outcomes of collective bargaining in the form of voting, or partaking in any voting scheme in the form of bargaining—the scope of negotiations \([\xi_1, \xi_2]\) brings the voting and bargaining schemes under one roof, both of which can be enriched by adopting this approach. Our initiative could also serve to unify the theoretical structure of economic analysis of productivity problem. Indeed, reg. Leibenstein [38], Altman [39], 2006, p. 149, wrote:

Leibenstein (1979, 493) argued that there are two components to the productivity problem: one relates to the determination of the size of the pie, while the second relates to the division of the pie. Looked upon independently, all agents can jointly gain by increasing the pie size..."the situation need not be a zero-sum game. Tactics that determine pie division can affect the size of the pie. It is this latter possibility that is especially significant.

Recall the main points of the bargaining theory. We will illustrate an efficient solution \((x^*, y^*)\) by 1S division maximizing the product of actors' payoffs above the disagreement point \(d = \langle d_1, d_2 \rangle\): 

\[
(x^*, y^*) = \arg \max_{0 \leq x, y \leq 1} f(x, y, \alpha) = (u(x) - d_1)^\alpha \cdot (g(y) - d_2)^{1-\alpha}.
\]

Although game theory purists might find the solution clear, the questions asked by many often include: What are \(x, y, \alpha, u(x), \) and \(g(y)\)? What does the point \(\langle d_1, d_2 \rangle\) mean, and how is the \(\arg \max\) formula used? The simple answer, known since Kalai [5],1977, as the asymmetric variant of Nash [11] problem, is as follows:

- \(x\) is the 1st actor's share of the 1S, with \(\alpha\) as the 1st actor's asymmetric power indicator, \(0 \leq x \leq 1, \ 0 \leq \alpha \leq 1\);
- \(u(x)\) denotes the 1st actor's payoffs of the 1st actor's 1S share \(x\);
- \(y\) is the 2nd actor's share of the 1S, where \(1 - \alpha\) is the 2nd actor's asymmetric power indicator, \(0 \leq y \leq 1\);
- \(g(y)\) denotes the 2nd actor's payoffs of the 2nd actor's 1S share \(y\).
Based on the widely accepted nomenclature, we refer to \( s = (u(x), g(y)) \) as to the utility or payoffs pair. The disagreement point \( d = \langle d_1, d_2 \rangle \) thus represents the payoffs the two actors collect if they cannot agree on how to share the wealth-pie. In the same vein, \( d = \langle d_1, d_2 \rangle = \langle 0, 0 \rangle \) represents the disagreement or breakdown point, whereby the players collect nothing.

A variant of asymmetric problem in our study emerges due to the fact that we succeeded to hide the arguments \( x \) and \( y \) in all functions, inclusive taxes \( \tau(\xi, x) \). The question of whether solution \( \xi^* \in [\xi_1, \xi_2] \) is efficient can thus be answered by solving the problem hiding the shares \( (x, y) \). We handle later an analytical solution exploiting payoffs in the form \( (u(\xi), g(\xi)) \) and taxes in the form \( \tau(\xi) \) within the scope of negotiations \([\xi_1, \xi_2]\) comprising the endpoints of the interval \([\xi_1, \xi_2]\). Akin to the above, political bargaining in our wealth-pie workshop can now be expressed by solution \( \xi^* \) maximizing the product of political payoffs above the threat point \( d = \langle d_1 = u(\xi_1), d_2 = g(\xi_2) \rangle \):

\[
\xi^* = \text{argmax}_{\xi \in [\xi_1, \xi_2]} f(\xi, \alpha) = (u(\xi) - d_1)^\alpha \cdot (g(\xi) - d_2)^{1-\alpha}.
\]

Unlike the traditional threat point \( d \), the public/vital goods amount \( d_2 \) in the game—the \( d_2 \) component of the point \( d \)—might be negative. This will apply in our study of a breakdown of negotiations, whereby borrowing or creating money is needed to balance the books for accounting welfare expenses—a situation of "genuine negative taxes". In the theory of public finance, the use of genuine negative taxes is not prohibited.

### 3.3. Pre-equity of breakdown

Beyond the asymmetric dynamics, the game inherits a premature disagreement or breakdown point, similar to that discussed by Osborn and Rubinstein [10], 1990, p. 72:

"We can interpret a breakdown as the result of the intervention of a third party, who exploits the mutual gains. A breakdown can be interpreted also as the event that a threat made by one of the parties to halt the negotiations is actually realized. This possibility is especially relevant when a bargainer is a team (e.g. government), the leaders of which may find themselves unavoidably trapped by their own threats."

In our game, the asymmetric solution incorporates the left-right wing political power indicators \( (\alpha, 1-\alpha) \) into a breakdown policy. In order to be addressed properly, the indicators cannot be given exogenously. To overcome this obstacle, we supply the game with a policy of endogenously extracted breakdown \( d = \langle d_1, d_2 \rangle \), on a condition referred to as the pre-equity of breakdown.

Traditionally, in the alternating-offers' game, the breakdown corresponds to two standard pairs of payoffs \( \{\langle 1, 0 \rangle, \langle 0, 1 \rangle \} \), or in the words of Osborn and Rubinstein [10], 1990, p. 73, "to the worst outcome." In the left-right political bargaining, due to the implicit pressure from the voters, as both political aim to find—at least from their perspective—a just and fair solution, there will always be a temptation for binary voters to defect to the other side. This puts the negotiations at risk \( 0 < q << 1 \) of
a premature collapse. Even under the worst circumstances, the quality and the size of the wealth-pie in the event of collapse should be equal for both politicians. This premise holds in these unfavorable circumstances as the entire pie will be decided upon by one of the politicians. Therefore, when the premature collapse occurs, it is important to arrange the terms of contract in such a way that neither politician can exploit or misuse the collapsed environment to its own advantage. To meet this condition, when normalizing the standard breakdown under the description valid for the alternating-offers’ game $\Gamma(q)$, we are working toward an endogenous form for equity in accordance with political non-conforming expectations.

As stated, the standard case of breakdown in the alternating-offers’ game corresponds to two pairs $\{(10), (0,1)\}$ of payoffs. In this form, the breakdown is generally found using ex-ante linear transformation—namely, the exogenous normalization of utilities. When the collapse is looming closer, the standard breakdown exposes political equity. Unlike the standard case, once the most unfavorable result might occur in reality, the resulting collapse must include additional parameters—the tax $\tau$ and the wealth $W$. In order to equalize—endogenously normalize—the breakdown, the politicians involved in negotiations can make a priori arrangements, or sign binding agreements upon these two parameters $\tau$ and $W$. However, without availability or warranty of such a pre-equity, an endogenous normalization is unrealistic. Therefore, in the view of the voters’ electoral maneuvering (discussed in the next subsection), even if the pre-equity normalization is not always achievable, it is more constructive to determine the breakdown according to some rational context.

Before proceeding further with a detailed assessment of the aforementioned definition, we recall the concept of wealth amount $W$, redistributed by the State as the average taxable income per capita narrowly defined as "prosperity or a commodity". Next, according to the conditions characterizing the collapsed environment, at the start of the negotiations, the draft of a contract covers taxes $\tau$ and—in line with our nomenclature—the wealth amount $W$. The product $\tau(\xi) \cdot W(\xi)$ identifies the size $z$ of the wealth-pie within an interval $[\xi_1, \xi_2]$—within the scope of negotiations, thus establishing the boundary for the two politicians. The lower limit, $\xi_1$, denotes the initial proposal, which is the most attractive for RWP, while being the most unattractive for LWP. In the same but inverse order $u_2 = u(\xi_2)$ can be paired with $g_2 = g(\xi_2)$. Having set these limits, we can proceed with examining how the breakdown $\{(u_1, g_1) : (u_2, g_2)\}$ might be conditionally, albeit endogenously, encoded into the game.

Indeed, we now contribute to implementing our wealth definition of how the breakdown can be established endogenously. To do so, we consider a situation driving the welfare policy in the context of cost-benefit equity. When the collapse of negotiations is looming closer, the differences in the amounts of wealth and taxes for funding low-cost welfare policy $\xi_1$ against an expensive policy $\xi_2$, $\xi_1 < \xi_2$—that is, funding payoffs $\langle u_1, g_1 \rangle$ for $\xi_1$ against $\langle u_2, g_2 \rangle$ for $\xi_2$, $u_1 < u_2$, $g_1 > g_2$—can amplify misunderstandings and contribute to traps. At the endpoints of the scope $[\xi_1, \xi_2]$, the wealth-pie sizes $z(\xi_1)$ and $z(\xi_2)$ at poverty lines $\xi_1$ and $\xi_2$ can require the delivery of wealth amounts $W(\xi_1)$ and
W(ξ₂), albeit at different prices, represented as taxes τ(ξ₁) and τ(ξ₂), c.f., Buchanan, [40], 1967, 4.7.1. Hence, prior to the start of the game, following cost-benefit equity, in the most adverse circumstances, the payoffs s₁ = ⟨u₁, g₁⟩ and s₂ = ⟨u₂, g₂⟩ should preserve equal prices τ for the delivery of equal amounts W of wealth. Such a market-driven interpretation of commodities delivery to the end destinations relies heavily on the size of the wealth-pie, which equals τ · W. It should be noted that this interpretation is only relevant to the situation of flat (proportional) taxes.

To explicate the interpretation of reasoning in previous lines, it is worth examining the "well defined bargaining problem," depicted as the contract curve in Figure 5. Based on the discussion so far, our goal is to fix an interval [ξ₁, ξ₂] solving two non-linear equations, τ(ξ₁) = τ(ξ₂) and W(ξ₁) = W(ξ₂), by attempting to find a cross-point (τ*, W*) where the curve crosses its own contour, as YX-axis coordinates, on the plain with (τ, W)—the roots ξ*₁ and ξ*₂. Although the calculus of the point (τ*, W*) is a high school exercise, it does not confirm the possibility of normalization in general. However, this does not invalidate our discussion, as we do not claim that the equity condition can be reached in all circumstances. It should still be pointed out that, in a number of examples where the validity of the condition was detected, we found a breakdown endogenously encoded into the game indicating normalization in the form of \((\xi*₁, g(\xi*₁)), (\xi*₂, g(\xi*₂))\).

In accord with the above, as the aim is to bring the politicians, if possible, into just and equal positions prior to negotiations, equalizing taxes τ and wealth amounts W in the collapsed environments ξ₁ and ξ₂ might be a rational starting point. Endogenously encoded into the game, we label the equity condition, [τ(ξ₁) = τ(ξ₂), W(ξ₁) = W(ξ₂)] as a pre-equity of breakdown. If valid, this condition equalizes fiscally realistic and just demands for public spending prior to negotiations—in particular, the size of the wealth-pie \(z(ξ₁) = z(ξ₂)\).

3.4. Political power design, to be continued in Section 6

Only the voting results can reveal the true incentives of people that will give the democracy its final judgment. The voting process is the only avenue for the voters to assume the roles of current or upcoming politicians to whom the opportunity will be granted in accord with people's wants to redesign the rules and norms of wealth redistribution. Voters' inequalities, life plans, background, social class and experience, native endowments, political capital, etc., determine the bulletin collected at the voting table. Consequently, voters' reasonable disagreements or interpretations of reality affect the individual choices and thus the voting results, thereby impacting political pre-election campaign. Voting results are not fully predictable due to the deviations in voters' views and opinions on how the wealth redistribution ought to be organized. The problem stems from the fact that welfare policy proposals that benefit minority of citizens sometimes require higher taxes. However, voters' majority, would be primarily guided by selfish attitudes toward lower taxes, which would implicitly affect the political bargaining positions. Such an attitude deserves, perhaps, a critical examination. Given these arguments, our question is: Why should the left-right wing politicians care about lower taxes?
It is therefore timely to recall political outmaneuvering with an implicit risk $q$, $0 < q << 1$, upon negotiations suffering a premature collapse. Indeed, Figure 5 depicts the contract curve of efficient public policies/proposals $\xi$ upon poverty lines in the bargaining game $\Gamma(q)$. Politically rational and economically effective proposals $\xi$, forming the curve, have been projected onto the two-dimensional space of the tax rate $\tau(\xi)$ and taxable income—the wealth amount $W(\xi)$. Although the payoffs $\langle u(\xi), g(\xi) \rangle$ are embedded in each point, the payoffs are not visible on the graph. These invisible/hided payoffs, in the upper part of the graph symbolize wealth-pie division $(x, y)$ into lower basic $x(\xi)$ yet higher of public goods shares $y(\xi)$ as left-wing wants $u(\xi)$ and the right-wing political objective $g(\xi)$ accordingly. Similarly, the payoffs in the lower part symbolize a reverse situation—the higher basic, vs. lower public goods, c.f., Figures 1-2. Thus, once all views are represented, the political payoffs $\langle u(\xi), g(\xi) \rangle$ for pledged tax hikes $\tau(\xi)$ are more favorable for some coalitions of voters compared to others. As voters’ preferences for the balance between basic and public goods vary, the approach to determining efficient poverty line resulting from eventual agreement between politicians is two-fold. Indeed, unless the tax hikes are excessively high, the upper coalitions’ representatives will always try to outmaneuver the lower coalitions’ representatives. The politicians are aware of this dynamic when taxes are high. As they feel trapped in negotiations, the result might be that binary voters defect to other side, putting the negotiations at risk $0 < q$ of premature collapse. In contrast, when taxes are sufficiently low, the range of eventual voters’ electoral maneuvering will shrink or even vanish. Therefore, the lowest tax is likely the one of desirable outcomes for the majority of citizens.

In line of reasoning that concerns the majority of citizens, it is appropriate to address in conclusion our main thesis of political power indicators $(\alpha, 1 - \alpha)$ design. Considering the bargaining outmaneuvering of left-right wing politicians according to the alternate offers' game $\Gamma(q)$, we state that the politicians on the opposite sides of the bargaining table might disagree with respect to the terms of outcomes, in which situation they would delay the decision while consolidating a draft of a consensus document. This document might not necessarily yield the best outcome for the citizens. From those citizens' perspective, representing the majority, the policy that minimizes taxes is always the most desirable choice. However, despite knowing the negative attitude toward higher taxes of the majority, the minimum tax rate might not necessarily be a desirable outcome in political view. Thus, politicians may choose to disregard the majority interests because political power of LWP or RWP, as rational actors/politicians, might be strong enough to negotiate selfish decisions favorable for both politicians alone. In order to entice politicians and prevent them from making selfish decisions, resulting in ultimate collapse in the negotiation process, their political power indicators $(\alpha, 1 - \alpha)$ ought to represent a natural power consensus benefiting them to choose a desirable outcome for themselves and for the majority of citizens—a platform that would be better to look-for and design in advance. This completed our preliminary investigation of the problem.
4. Fiscally safe welfare policies, continued from 3.1

The delivery of basic goods, which counteracts negative contingency, if it occurs, is the left-wing main political responsibility. Herewith, the left-wing political intervention is of the greatest political importance. It is universal in the sense that it pertains to all citizens, regardless of one's situation before or after the contingency. Under this premise, basic goods that are available to citizens are of sufficiently high quality and poverty is not allowed, as stressed by Greve [41], 2008, p. 58. This course provides a relatively high level of welfare spending and taxes, creating misbalance in the books accounting for public finances, i.e., introducing volatility conditions into the wealth-pie delivery. Hence, secured largely independently of market forces, the high level of basic goods might have a conflict-driven effect on the welfare policy, which should not be borne by citizens alone as, as already noted, the state has a duty to help the disadvantaged.

Assumed that the conflict-driven welfare policy guided our political actors in trying to sign an agreement, the left-wing politicians aimed to secure an efficient size of the wealth-pie. Thus, LWP prescribed the size of the pie and proposed the division method, which the right-wing politicians accepted or rejected. If rejected the RWP suggested their own understanding of division, while only having the authority to recommend a size that the LWP might not be obligated to accept. We also assumed that, upon delivery to its end destinations, the wealth-pie remained fiscally safe, i.e., not changing its size. Under the rules of the alternating-offers' procedure (see later), the game will continue until a consensus is reached. This, however, presupposes that left-wing politicians are committed to the share, albeit without being committed to the size.

Let us now envisage a contrasting scenario, whereby the public spending increases. Thus, both actors know that, upon delivery, the size of the wealth-pie might change. This, in turn, leads to a misbalance between the subsidies and tax-revenue, which can put the pie size in doubt or make it even fuzzier. The difficulty related to political pledges might thus force both sides to retreat. In such volatile conditions, the wealth-pie is no longer fiscally safe and might affect the expectations of both politicians. Therefore, a fiscally safe plan in spite volatile conditions for delivering and division of the wealth-pie is needed. Otherwise, unless welfare policy fails to enforce fiscal safety, the rules and norms of the LI are not living up to their claims. Thus, having a criterion for determining whether a welfare policy is fiscally safe is necessary.

It is helpful to focus first on welfare policy without any warranty of fiscal safety. It could, for example, be determined by the poverty line $\xi$, identifying the recipients of wealth redistribution. When $\xi$ is low, the variable $\sigma$, $0 < \sigma \leq \xi$, allocates the income of the needy or the benefit claimants. In this scenario, the benefit claimant $\sigma < \xi$ claims and receives a benefit or subsidy proportional to $\xi - \sigma$, i.e., $r \cdot (\xi - \sigma)$, as previously discussed, while all other citizens—both the wealthy and those with marginal income, denoted as $\sigma > \xi$ and $\sigma = \xi$, respectively—receive a zero subsidy.
Next, we study a specific scheme highlighting the readiness of the society to fund welfare and public spending. For this analysis, we assume that the average cost \( \mathbf{B} \) of the subsidies and the average taxable income \( \mathbf{W} \) both depend on the poverty line parameter \( \xi \), \( \mathbf{B} \equiv \mathbf{B}(\xi) \), \( \mathbf{W} \equiv \mathbf{W}(\xi) \) —this is realistic, as shown in Appendix A1. As previously narrowly defined, \( \mathbf{W}(\xi) \) can refer to the wealth amount. Based on our perception of incomes \( \sigma \) density \( \mathbf{P}(\sigma, \xi) \) distribution samples, the product \( \tau \cdot \mathbf{W}(\xi) \) estimates the average tax-revenue. Let the average cost of public goods be \( g(\xi) \), whereas the size \( \mathbf{z}(\xi) \) of the wealth-pie equals \( \tau \cdot \mathbf{W}(\xi) \), \( \mathbf{z}(\xi) = \tau \cdot \mathbf{W}(\xi) \). We assume that welfare and public spending reached its recipients—namely, the total spending equals \( \tau \cdot \mathbf{W}(\xi) = \mathbf{B}(\xi) + g(\xi) \). This suggests that the basic and non-basic goods have been delivered to their final destinations. In other words, the wealth collected through tax channels is spent.

Now, let us assume that politicians in the game preferred to commit to the shares fixing \((x, y)\), and might agree to hold the balance \( \mathbf{B}(\xi) = x \cdot \tau \cdot \mathbf{W}(\xi) \) of the books accounting for financing the subsidies \( \mathbf{B} \). That is, the left-wing politicians must be ready to finance the subsidies, i.e., to deliver \( \mathbf{B}(\xi) \) by division the wealth-pie \( \tau \cdot \mathbf{W}(\xi) \). The politicians pledged to keep the balance \( \mathbf{B}(\xi) = x \cdot \tau \cdot \mathbf{W}(\xi) \) of the subsidies between credits \( \mathbf{B}(\xi) \) and debts \( x \cdot \tau \cdot \mathbf{W}(\xi) \) as a portion \( x \) of the wealth-pie \( \tau \cdot \mathbf{W}(\xi) \). The balance also specifies the welfare policy \( \xi \)—an implementation of the poverty line \( \xi \), welfare reform, pact, program, etc. While the balance is initially valid, it might not be in the future, putting the adjustment in \( \xi \) on the agenda either once or repeatedly. Thus, the policy \( \xi \) might represent a problem of fiscal imbalance. However, almost all citizens—even if for different reasons—will prefer the opposite in the long run: a fiscally safe policy \( \xi \). For this reason, we now shift the focus on examining a constraint that corresponds to fiscal safety of welfare policy \( \xi \), identifying—what we called above as idempotent—the safe delivery of the wealth-pie to its end destinations.

4.1. Idempotent rules and norms

The delivery of basic and public (non-basic) goods does not necessarily safeguard the funding of the expenses. As the expenses neither match nor prevent taxation hikes, the size of the wealth-pie could vary too rapidly. This leads, as previously discussed, to numerous adjustments of welfare policy rules and norms. To mitigate this issue, we have to look at the sequence \(.\xi', \xi".\) of multiple adjustments of the poverty line \( \xi \). This highlights the fact that, on delivery, the adjustments of the wealth-pie are undesirable. Consequently, it is better to keep the size of the pie unchanged, i.e., fiscally safe. In other words, when replacing the old policy \( \xi' \) with \( \xi" \), the two must coincide. Similar schemes, known as idempotent, stem from bounded rationality mechanisms. This suggests that, even if welfare policy rules and norms are subject to multiple adjustments, this should not change the machinery of subsidies' payments. In particular, when implemented twice, the rules must produce the same outcome. Therefore, to guarantee the fiscal safety of the poverty line, such an understanding requires that the poverty lines must coincide amid a sequence of pairs \((\xi', \xi")\) at some matching policy \((\xi' = \xi")\).
The need to balance the books accounting for the delivery of subsidies $B(\xi) = x \cdot \tau \cdot W(\xi)$, in spite the wealth-pie volatility, can also be seen as immunity for financing the welfare policy. In particular, the immunity holds down, or at least realistically limits the h-effect of wealth redistribution. Given the immune, i.e., fiscally idempotent, composition $[B(\xi), W(\xi)]$, the idempotent scheme is equivalent to implementing the policy $\xi$ only once. For this reason, we assume that the rules and norms of the subsidies' payments have been socially planned and redesigned accordingly.

In this idempotent mode that outlines the fiscal safety of public spending, the rules and norms must reflect idempotent policy $\xi$ that brings the spending policy into focus. We therefore conclude that the expenses $x \cdot \tau \cdot W(\xi)$ designated for welfare spending must be in balance not only for funding subsidies $B(\xi)$, when the particular policy $\xi$ takes effect, but also the policy $\xi$ must enforce the fiscal safety in the full spectrum of current and future events.

Clearly enough, the balance $B(\xi) = x \cdot \tau \cdot W(\xi)$ is a static relationship leading to functional dependency $\tau \equiv \frac{B(\xi)}{x \cdot W(\xi)}$ links the arguments $\xi$ and $x$. Hereby, the tax rate $\tau$ becomes a function of $\xi$ and $x$, expressed as $\tau = \tau(\xi, x)$. The post-tax residue $\pi(\xi, \tau) = (1 - \tau) \cdot (\xi - \phi) + \phi$ of the marginal citizens' $\sigma = \xi$ comprises fiscal limitations of wealth redistribution according to rules and norms in force of subsidies payments; $\phi$ determines the personal allowance parameter, see above. Therefore, the dependency $\tau = \tau(\xi, x)$ transforms $\pi(\xi, \tau)$ into fiscally realistic social position $\pi(\xi, \tau(\xi, x))$. However, irrespective of the current expenditure on basic goods, the real cost of living does not necessarily match $\pi(\xi, \tau(\xi, x))$. Hence, ensuring realistic and fiscally idempotent rules and norms, and/or, in particular, attempting to avoid the h-effect of this mismatch or adopt rules to keep the effect tolerable at the least, an equation for a fiscally idempotent policy $\xi$ should be solved.

**Observation 1.** Constraint on left-wings political wants $u = \pi(\xi, \tau(\xi, x))$ is the necessary to uphold idempotent fiscal rules and norms of imposed budget-constraint $B(\xi) = x \cdot \tau \cdot W(\xi)$.

The observation claims that, whatever tax increase is implemented, the poverty line residue $u$ of the marginal citizens' $\sigma = \xi$ is unfeasibly high to be reached when the condition has been violated.

**Corollary.** When $u = \pi(\xi, \tau(\xi, x))$ solves for $\xi$ the subsequent adjustments $\xi', \xi'',...$ are unnecessary. An option to change their welfare positions is irrational for citizens with incomes $\sigma < \xi$ or $\sigma > \xi$—the root $\xi$ holds down (realistically limits) the h-effect. Appendix A3 includes all proofs.

The fiscally idempotent policies $\xi$ induce the basis for solutions in our game as fiscally idempotent compositions $[B(\xi), W(\xi)]$. A reasonable question thus emerges: Which taxable income $W(\xi)$ characterizes fiscally idempotent welfare policies $\xi$ for the delivery of subsidies $B(\xi)$? The answer is included in the following three constraints:

1 Below, we continue to refer to the average taxable income as “wealth.”
Delivery-constraint by which the wealth-pie is spent—the basic and public goods have been delivered. This form of constraint makes sense only for proportional or flat taxes. Flat taxes will later substantially simplify the method of function minimization with constraints.

\[
\tau \cdot W(\xi) = B(\xi) + g
\]  

(1)

Budget-constraint imposed on subsidies finance in accordance with the share \( x \) of the wealth-pie—the tax-revenue. The left-wing politicians promise to credit/debit the account \( B(\xi) \) that must be equal to the average of subsidies shifted by the policy \( \xi \).

\[
B(\xi) = x \cdot \tau \cdot W(\xi)
\]  

(2)

Stability constraint that determines fiscally idempotent property of (2). In contrast to \((\sigma, \tau) \in \mathbb{R}^2\), we distinguish poverty line residues \( u = \pi(\xi, \tau) \) as one-dimensional curves \( \pi(\xi, \tau) \in \mathbb{R} \subset \mathbb{R}^2 \).

\[
\phi, u, \xi \Rightarrow z, x, \alpha, \tau, \langle u, g \rangle
\]

Taking the expression \( \tau(\xi, x) = \frac{B(\xi)}{x \cdot W(\xi)} \) out of the constraint (2) and replacing \( \frac{B(\xi)}{x \cdot W(\xi)} \) into \( u = \pi(\xi, \tau(\xi, x)) \), the constraint given in (3) can be resolved with a fiscally idempotent policy for \( \xi \):

\[
L(\xi, u) = (\xi - \phi) \cdot B(\xi) - x \cdot (\xi - u) \cdot W(\xi) = 0
\]  

(4)

Referred to as the volatility-constraint, the constraint (4) determines the fiscal safety module. It holds down the h-effect amalgamating the constraints (2) and (3) by balancing the books accounting for subsidies.

**Summary.** The outcome \( \phi, \xi \Rightarrow z, x, \alpha, \tau, \langle u, g \rangle \) constitutes the citizens’ bargaining shield for wealth redistribution that relates to a bundle of arguments or constants: \( \phi, \xi \) are controls, and \( z, x, \alpha, \tau \) are status variables. \(^2\) While \( \langle u, g \rangle \) are the competing political proposals:

- \( \phi \) – the personal allowance establishing the tax bracket \( [\phi, \infty) \); it is an ex-ante control (tuning) variable, \( 0 < \phi = \text{const} < \xi \);
- \( \xi \) – the income frame, the poverty line; a policy determining who is living in poverty, as well as the choice or the control parameter;
- \( z \) – the size \( z = \tau \cdot W(\xi) \) of the wealth-pie; the account of wealth-pie that equals public spending per capita when taxes are proportional;
- \( x \) – the share of wealth-pie of size \( z \); a portion \( x \) of \( z \) to be deposited in favor of the left-wing politicians for funding the subsidies, \( 0 \leq x \leq 1 \);
- \( \alpha \) – the political power of the left-wing politicians, \( 0 < \alpha < 1 \);
- \( \tau \) – the marginal tax rate, the rate \( \tau(\xi, x) \) of the wealth amount \( W(\xi) \) determined by (1);
- \( u \) – the after-tax residue of the income frame equal to the poverty line \( \xi \), the wants function \( u(\xi, x) \) of the left-wing politicians as determined by (2) and (3);
- \( g \) – the objective’s function \( g(\xi, x) \) of the right-wing politicians, determined by (1) and (2); the account for the refund of public goods expenses per capita.

The share \( x \) and the marginal tax rate \( \tau \), due to the constraints 1 through 3, become functions of arguments \( \xi, g : x \equiv x(\xi, g) \) and \( \tau \equiv \tau(\xi, x(\xi, g)) \). This form of dependence appears next in the module of alternating-offers’ bargaining game.

\(^2\) Status and control variables are the prerogatives of control theory.
5. Alternating-offers in the left-right political bargaining, continued from 3.2

Suppose that politicians, resulting in commitments of how to share fairly the wealth-pie, agreed to play the alternating-offers’ bargaining game $\Gamma(q)$, [10], 1990, p. 31. In doing so, rational politicians are motivated to align the procedure to participate in any eventual agreement. The risk $q > 0$ of a premature collapse during negotiations, especially early in the game, might be the driving force behind their commitment to reach the consensus. Once a consensus on division is reached, they must agree on who will make the decision about the size of the pie. Politicians negotiate on such matters when there are equal and symmetric preconditions in place that guarantee their equal rights. Thus, both will play an equal role in the decision regarding the size. However, considering the right-wing vital political objective of wealth redistribution, it will be realistic to reduce the scope of RWP’s duties concerning welfare matters, while allowing them to retain their advisory rights. We proceed as follows.

5.1. Bargaining procedure

We stressed that, in a representative democracy, the division of the wealth-pie will always be subject to controversy. Recall that we consider two politicians—one acting in the role of LWP, who is struggling for basic goods, and the other, representing RWP, advocating for non-basic goods. A precondition for the bilateral agreement is that the expectations of these two politicians depend solely on efficient policies of the LWP within the framework of how to set the poverty line $\xi$. However, politicians are more concerned with shares $(x, y)$ than with the size of the wealth-pie. As a consequence of this independence, efficient poverty line $\xi^*$ provide shares connected with efficient divisions $(x^*, y^*)$. Accepting this precondition, the RWP will only propose efficient lines $\xi^*$. Failure to do so would result in all other shares being rejected with certainty. Nonetheless, it is realistic that the RWP would—by negligence, mistake or some other reason—recommend an inefficient poverty line $\xi'$, which the LWP would mistakenly accept. It is also possible that, in a reverse scenario, the LWP would choose to disregard an efficient recommendation $\xi^*$. This will be irrational handling as, in any agreement, regardless of the underlying motives, both politicians are committed by proposals to shares $(x, y)$. Indeed, within the scope of negotiations $[\xi_1, \xi_2]$, the recommendation $\xi^*$, consolidates with efficient RWP’s share proposal $y^*$. While accepting $1-y^*$, but shifting its own LWP’s $\xi^*$ mistakenly to $\xi' \neq \xi^*$, at which both politicians must be committed to $(x^*, y^*)$, the shift $\xi'$ becomes inefficient—superfluous. Thus, making a proposal, the RWP’s recommendation on poverty lines makes a rational argument that the LWP must accept or reject in a standard way. Such an account, as we believe, explains that the outcome of the bargaining game might be a desirable poverty line $\xi \in [\xi_1, \xi_2]$. Hereby, the interval, referred to as the scope $[\xi_1, \xi_2]$ of negotiations, bids proposals, which now, by default, are linking efficient lines $\xi^*$ with shares $(x^*, y^*)$. The bargaining occurs exclusively in the interval $[\xi_1, \xi_2]$ as a scope for efficient lines $\xi^*$ of most trusted policy platforms for negotiations, where both players would choose, accepting or rejecting, the proposals. Political competition, depending on $[\xi_1, \xi_2]$, arrange a contract curve $S_b$ (shown in Figures 4-5) as a way to assemble the bargain portfolio. Given that the portfolio "has changed its color from shares to lines," the politicians can now conceive themselves as making poverty line proposals. If a proposal is rejected, the roles of politicians change and a new proposal is submitted. The game continues in the traditional way by alternating offers.
5.2. Analysis

We now proceed to a more accurate analysis of the game rules. Although the rules can be perceived as fiscally idempotent, the game itself contains a new challenge. The elevated poverty line $\xi$ does not necessarily increase the marginal citizens' $\sigma = \xi$ after-tax residue $u(\xi, x)$. The low-income citizens—the benefit recipients—can claim subsidies whereby an increased number of claims might have a reverse effect that declines $u(\xi, x)$. Indeed, in contrast to increasing poverty line $\xi$ and despite the required unavoidable increase in taxes—as the hazard (h-effect) is still present—this will decrease the residue $u(\xi, x)$. With the proviso that the left-wing politicians commit to the share $x$, the right-wing politicians are left with $y = 1 - x$. Thus, the fiscally idempotent poverty line tax residues $u(\xi, x)$ correspond to a more narrow set than $0 \leq x \leq 1, 0 \leq y \leq 1$—the set of shares $(x, y)$ of what we refer arrange a contract curve $S$ of payoffs $((u(\xi, x), g(\xi, y))$ with poverty line $\xi$ as a parameter.3

Assuming that the maximum of a single $\cap$-peaked residue function $u(\xi, x)$ can be reached, the peak position $\xi^* = \arg \max_{\xi} u(\xi, x^*)$ indicates an efficient welfare policy. Although the bargain portfolio of left-wing politicians contains an efficient policy $\xi^*$ as a function of $x^*$, the portfolio also contains the share $x = x^*$. The top value given by $u = u^*$, in the inverse situation, which corresponds to $u^*$, consolidates an efficient policy $\xi^* \in [\xi_1, \xi_2]$. A unique share $x^*$, which solves $u(\xi^*, x) = u^*$, and corresponds to $g(\xi^*, y^*) = g^*$, represent the non-conforming expectations of politicians. We can thus refer to the shares $(x^*, y^*)$ as an efficient division connected with the policy $\xi^*$. Depicted in Figure 4 in various projections—on payoffs $(u^*, g^*)$, and in Figure 5 on wealth amount $W$ and taxes $\tau$—efficient peaks $\xi^*$, which correspond to efficient shares $(x^*, y^*)$ geometry. This geometry highlights the top values $u^*$—namely, efficient policies of left-wing politicians at peaks $\xi^*$ that refers to the well-known result of Canto et al., [42], 1981, p. 11—the Laffer curve:

"The marginal tax-revenue raised decreases with increase in tax rates, finally reaching some point where the marginal tax-revenue raised is zero. Beyond this point, any tax rate increases will reduce revenue collection."

Our result pertaining to the single-peaked wants of the left-wing politicians is similar. First, "poverty line residue $u$ being proposed in the normal range of poverty line parameter $\xi$." Next,

"...by passing through the top point of $u$ as a function, the proposals $u$ will be assessed and reviewed in the range of prohibited values of $\xi$."

We previously introduced an idempotent composition $[B(\xi), W(\xi)]$—the average $B(\xi)$ of the subsidies, and the average $W(\xi)$ of the taxable income—the wealth. The expectations of the two politicians, reflecting their preferred rules and norms pertaining to subsidies, can now be set using the composition $[B(\xi), W(\xi)]$. At the end of the subsection, the composition will lead to an appropriately settled bargaining problem that will associate the threat originating from the implicit partaker—in the form of the electoral maneuvering of voters—with an implicit risk of the negotiations collapsing prematurely. This requires adding two rigorous suppositions to the standard rules of the game we have already presented. Let us first specify the payoffs.

3 We stressed already the worsening quality of welfare services for all citizens when the LI-level is “climbing” high.
Political payoffs of 1st/2nd actors and the third partaker’s implicit risk factor are defined as follows:

Politician No.1, \( u \) – the left-wing political wants, the marginal citizens’ \( \sigma = \xi \) after-tax residue, basic necessities of the needy, cost of living;

Politician No.2, \( g \) – the right-wing political objective, expenses that benefit all citizens—expenses without subsidies;

Third Partaker, \( q, \tau \) – voters’ electoral maneuvering facing higher taxes \( \tau \) expressing an implicit risk \( 0 < q << 1 \) of the negotiations collapsing prematurely.

As promised, we now assume that the rules and norms of the wealth redistribution that are efficient with respect to the wealth-pie division include the volatility-constraint \((4)\), which certifies the idempotent composition \([B(\xi), W(\xi)]\) for the policy \( \xi \). In the game, the composition \([B(\xi), W(\xi)]\) could not be implemented without the volatility-constraint \(L(\xi, x, u) = 0\) (Observation 1). This assumption is contingent on the conclusions of the previously undertaken analysis.

When varying \( \xi \) under their own rules and norms, let us assume that LWP propose a fiscally idempotent policy \( \xi = \xi' \), which—for each share \( x = x' \) they commit to—links \( x' \) to \( \xi' \), ensuring, irrespective of who originates the proposals \( x' \) or \( y' \). The efficient proposal of poverty line residue \( u(\xi', x') = \max_\xi u(\xi, x') \). Clearly, inefficient recommendation \( \xi' \), proposed by the RWP if \( \xi' \neq \xi' \) for share \( y' \), will be rejected by the LWP. As a result, an efficient policy \( \xi = \xi' \) must occur on contract curve amid efficient shares \( x' \) at \( \langle u' = u(\xi', x'), g' = g(\xi', x') \rangle \) as an ongoing precondition for the agreement—this procedure was discussed previously. Indeed, LWP have no reason to reject efficient recommendation \( \xi' \), as doing so, when \( \xi' \neq \xi' \), they cannot ultimately maintain the efficient commitment to \( x' \). Below, we assume the efficiency by default when it is convenient.

**Observation 2.** Idempotent policies \( \xi \) at the contract curve \( \mathbf{S}_\xi = \langle u(\xi, x), g(\xi, x) \rangle \), which certifies the composition \([B(\xi), W(\xi)]\), must satisfy the constraint

\[
D(\xi, x, u) = \frac{\partial}{\partial \xi} L(\xi, x, u) = \frac{\partial}{\partial \xi} [\xi \cdot (\xi - u) \cdot W(\xi)] = 0. \tag{5}
\]

Particularly, when we collated sub-expressions and introduced some simplifications upon

\[
\begin{align*}
Q(\xi, \tau, g) = 0 & \rightarrow \text{Delivery(1)}; \\
L(\xi, x, u) = 0 & \rightarrow \text{Volatility(4)}; \\
D(\xi, x, u) = 0 & \rightarrow \text{Contract(5)};
\end{align*}
\]

these constraints, with the proviso of flat taxes, together with the previously detailed preliminary settings \( \tau_\xi' > 0, \tau_\xi'' > 0, u'_\xi < 0, u''_\xi < 0, u'_\xi > 0, u''_\xi > 0, g'_\xi > 0, g''_\xi > 0, g''_\xi \neq 0 \), lead to an analytical solution:

\[
u(\xi) = \xi - \frac{(\xi - \phi)}{v(\xi)}, \text{ where } v(\xi) = 1 + (\xi - \phi) \cdot \left(\frac{B'(\xi)}{B(\xi)} - \frac{W'(\xi)}{W(\xi)}\right);^4 \quad \tau(\xi) = \frac{1}{v(\xi)}.
\]

\( ^4 \) Rates \( W'(\xi) \leq 0 \), \( W'(\xi) \geq 0 \) of the changes in the wealth amounts \( W(\xi) \) are essential for the analysis, whereas the function \( B(\xi) \) is valid only when \( B'(\xi) > 0 \), and \( 0 < \phi < u < \xi \).
\[ g(\xi) = \frac{W(\xi)}{v(\xi)} - B(\xi); \text{ the size of wealth-pie } z(\xi) = B(\xi) + g(\xi) = \frac{W(\xi)}{v(\xi)}. \]

Now it is evident that payoffs \( \langle u, g \rangle \) at the contract curve \( \mathcal{S}_b \) depend exclusively on policies \( \xi, \langle u(\xi), g(\xi) \rangle \in \mathcal{S}_b \). Therefore, we conclude that politicians are only concerned making proposals over efficient policies \( \xi \) since effective shares \((x, y)\) have been linked to \( \xi \). Contract curve \( \mathcal{S}_b = u(g) \) in Figure 4 illustrates the payoffs. The functions \( g(\xi) \) and \( u(\xi) \) in the form presented above are, in fact, not a subject to any constraints. They are mathematically derived in Appendix A4.

Before proceeding with further line of analysis, let us recall the threat phenomenon created by voters that increases the implicit risk of the negotiations collapsing prematurely. As noted previously, if politicians reject their counterpart's proposal—knowing that it is risky to continue the bargain—they will likely consolidate a draft. Voters might emanate a threat to vote against the draft when politicians, without fulfilling the voters' terms, try to continue bargaining over costly and controversial proposals, thereby putting the negotiations at risk of collapse, see above.

Suppose that politicians bargain over all fiscally idempotent policies \( \xi \in [\xi_1, \xi_2] \) within the scope of negotiations \([\xi_1, \xi_2]\). We follow the alternating-offers' game \( \Gamma(q) \) with an exogenous risk \( 0 < q << 1 \) of a premature collapse, as described in [10], 1990, pp. 71-76. We posit that, each time the proposal \( \xi \) is rejected by one of the politicians, the momentary phase of the game consolidates a draft, which can be opposed by the voters, as just recalled. In these circumstances, the politicians might be uncertain on how to proceed, if the voters' terms are not met. As a result, they might choose to leave the bargaining table prematurely. Extracted from the endpoints \( \xi_2 < \xi_2 \) of contract curve \( \mathcal{S}_b \), the outcome \( \{\langle u_1, g_1\rangle, \langle u_2, g_2\rangle\} = \{\langle u(\xi_1), g(\xi_1)\rangle, \langle u(\xi_2), g(\xi_2)\rangle\} \) naturalize this risk \( q \) in the worst-case.

What is known as the "well-defined bargaining problem," first introduced by Roth, [15], 1977, or the individual rationality associated with the Nash [11] bargaining scheme \( \langle \mathcal{S}, d \rangle \), seems to be instructive to look closer. Indeed, inequalities \( g_1 \geq g_2 \), \( u_1 < u_2 \) hold for the pair \( d = \langle d_1 = u_1, d_2 = g_2 \rangle \). Synthesizing the unfavorable political outcome \( \{\langle u_1, g_1\rangle, \langle u_2, g_2\rangle\} \) into a policy \( d \) on poverty introduced below will naturalize the Nash disagreement point \( d \) into the problem \( \langle \mathcal{S}_b, d \rangle, \mathcal{S}_b \subset \mathbb{R}^I \).

Thus, compared to the traditional approach of compact convex set \( \mathcal{S} \subset \mathbb{R}^2 \), inequalities \( s > d \) are also true for any pair \( s \in \mathcal{S}_b \). Therefore, the pair \( \langle \mathcal{S}_b, d \rangle \) for the contract curve \( \mathcal{S}_b \) becomes a well-defined bargaining problem. However, as whether the policy \( d \) is a fiscally idempotent outcome of the game is not immediately apparent, the following observation removes any doubt.

**Observation 3.** To test whether the point \( d = \langle d_1, d_2 \rangle = \langle u_1, g_2 \rangle \) becomes a fiscally idempotent outcome of the left-right political bargaining, it is necessary and sufficient that there exists a policy \( \delta \) on poverty, which solves the equation:

\[ (\delta - \phi) \cdot (B(\delta) + d_2) - (\delta - d_1) \cdot W(\delta) = 0; \text{ The condition } \delta \notin [\xi_1, \xi_2] \text{ must hold true.} \]
It should be noted that, in the worst-case $\delta$, the wealth redistributed in the society equals $W(\delta)$—the average of expenses for funding the subsidies equal $B(\delta)$—proposal $\delta$ depends on the endpoints of the bargaining interval $[\xi_1, \xi_2]$. This dependence, provided the Equation (6) can be solved for $\delta$, serves as the basis for validation of the pre-equity condition of breakdown, as discussed in Section 7.

**Observation 4.** In the alternating-offers’ game $\Gamma(q)$ with the risk $0 < q << 1$ of negotiations collapsing prematurely into the disagreement point $\langle d_1, d_2 \rangle$, the functions $(u(\xi) - d_1)^\alpha$ and $(g(\xi) - d_2)^{1-\alpha}$ imply bargaining payoffs of left-right wing politicians respectively. Thus, solving the equations $(1-q) \cdot (u(\lambda_1) - d_1)^\alpha = (u(\lambda_2) - d_1)^\alpha$ and $(1-q) \cdot (g(\lambda_2) - d_2)^{1-\alpha} = (g(\lambda_1) - d_2)^{1-\alpha}$ for variables $\lambda_1, \lambda_2$ (without proof), the solution $\lambda$ of the well-defined bargaining problem $\langle S_b, d \rangle$ is close to the pair $\langle \lambda_1, \lambda_2 \rangle$, $\lambda_1 \leq \lambda \leq \lambda_2$.

As explained by Osborn & Rubinstein, [10], 1990, p. 75, the outcome in our study of this bargaining game $\Gamma(q)$ encapsulates the power indicators $(\alpha, 1-\alpha)$ of the left-right wing politicians. In the next section, we consider design of political power indicators $(\alpha, 1-\alpha)$ using the solution $\lambda$ minimizing the tax burden with respect to an appropriately settled bargaining problem $\langle S_b, d \rangle$.

6. **Political power design**, continued from 3.4

We will elaborate on power indicators $(\alpha, 1-\alpha)$ specific further using bargaining scenario of $1S$ division, based on the previously discussed axiomatic approach— $\alpha$ signifies LWP’s political power, and $1-\alpha$ the political power of RWP, $0 < \alpha < 1$.

Considering the $$(x^*, y^*) = \arg \max_{0 \leq x+y \leq 1} f(x, y, \alpha) = (u(x) - d_1)^\alpha \cdot (g(y) - d_2)^{1-\alpha}$$ formula, a question emerges: What standard of $1S$ division will assist a moderator designing the power indicator $\alpha$ of 1st actor? What will facilitate 1st actor during to negotiations in obtaining a desired, or any other share $x^*$ of $1S$? To answer these questions, let us assume that 2nd actor might only accept or reject the 1st actor proposals. We can thus start redesigning the power indicators $(\alpha, 1-\alpha)$ by replacing $y = 1 - x$, and taking the derivative of the result $f(x, 1-x, \alpha)$ with respect to the variable $x$ by evaluating $f'_x (x, 1-x, \alpha)$. Finally, suppose for a moment that $x^*$ share of $1S$ is an allegedly desirable solution. With $x = x^*$, the equation $f'_x (x^*, 1-x^*, \alpha) = 0$ can be solved for $\alpha = \alpha^*$.

In general, one might feel comfort in the following egalitarian judgment:

To count on $x^*$ share of $1S$ is a realistic attitude toward the 1st actor’s position of negotiations. Indeed, even in the face of the fact that the 2nd actor might have, e.g., a stronger negotiating power than the 1st actor, $\alpha^* < 1-\alpha^*$, the 1st actor, sooner rather than later, might predict whatever the 2nd actor prefers. Therefore, the former 1st would have the latter 2nd agree to a concession.

When, e.g., the voters’ representatives attempt to redesign accordingly political power indicators to $(\alpha, 1-\alpha)$, we assume that politicians will try to share the wealth-pie in a $1S$ manner. In doing so, however, we suppose that both politicians are ready to proceed with tax concessions. Reflecting just illustrated axiomatic bargaining toward allegedly desirable $1S$ share $x^*$, we proceed as follows.
In accordance with our analytical solution without constraints, the contract curve \( S_b = u(g) \) corresponds to a curve \( \langle u(\xi), g(\xi) \rangle \). Moving along the curve while taking into account the scope of negotiations \([\xi_1, \xi_2]\), the expectations \( \tau(\xi) \) of voters’ majority lead to detection of \( \tau_{\min} \rightarrow \tau(\xi) \):

\[
\min_{\xi \in [\xi_1, \xi_2]} \tau(\xi) \bigg| \tau(\xi) = \frac{1}{v(\xi)}.
\]

With the proviso that \( \tau(\xi) \) is concave and smooth enough, the detection point of \( \tau_{\min} \) is the root \( \lambda \) of the equation \( \tau'(\xi) = 0 \). Consequently, akin to egalitarian judgment above, the root \( \lambda \) might help in redesigning the rules and norms of the wealth redistribution. This can be done, adjusting the \( \alpha \), in a way that the political power \( \alpha \) of the left-wing politicians will be sufficient to persuade the right-wing politicians to agree upon the poverty line residue \( u(\lambda) \).

Indeed, in the left-right political bargaining, the old standard (discussed above) of how to share the 1S can now be a new Standard pertaining to how to plan the wealth redistribution rules and norms. Under this premise, we can set \( f(\xi, \alpha) = (u(\xi) - d_1)^\alpha \cdot (g(\xi) - d_2)^{1-\alpha} \), where \( \alpha \) facilitates the political power of the LWP. Instead of \( x^* = x^* \), planning the rules, we suppose that \( \xi = \lambda \) is an allegedly desirable solution. Acting again, we first take the derivative of \( f(\xi, \alpha) \), with respect to \( \xi \) evaluating \( f'_\xi(\xi, \alpha) \), and then we solve the equation \( f'_\xi(\xi = \lambda, \alpha) = 0 \) for \( \alpha \). As a result, the root \( \alpha^* \) will correspond to the redesigned political power of the left-wing politicians. This is the result as it appears.

**Summary.** To control the left-right wing political agreement on shares \((x, y)\) of the wealth-pie, akin to the new Standard above, the majority of citizens can accept or reject a premature agreement archived at the moment during the negotiations, thereby voting for or against the division. Therefore, the majority will favor the policy \( \lambda \) that minimizes the tax burden. This restriction allows us to rebalance the welfare institutions or finance resources by appropriate design of power indicators \((\alpha, 1-\alpha)\) of the left-right wing politicians ensuring that the most favorable shares \((x^*, y^*)\) of the wealth-pie division would incorporate the Nash axiomatic—the minimum tax—solution \( \lambda \) into the bargain portfolio as the best outcome. This is our case study of tax policy in which only a minority would object to a proposal that corresponds to the tax rate minimum at the contract curve. In doing so, the implicit pressure of citizens will be lower. Therefore, to be implemented in favor for majority, the minimum appears to be a desirable consensus.

**Observation 5.** Given that politicians can reach a preliminary agreement on tax rate \( \tau = \tau(\xi) \), condition \( \lambda = \arg \min_{\xi \in [\xi_1, \xi_2]} \tau(\xi) \) is necessary to put forward a poverty proposal \( \lambda \) before voters by appropriately designing the power indicators \((\alpha, 1-\alpha)\) in advance. At the contract curve \( S_b \), the proposal \( \lambda \) outlines a unique outcome \( \phi, \xi \Rightarrow z, x, \alpha, \tau(\lambda), \langle u(\lambda), g(\lambda) \rangle \in S_b \).

**7. Discussion**

One possible way to reveal the true essence of the economic reality behind the left-right political bargaining could be determining whether it is true that funding subsidies and maintaining the budget in balance will be difficult to sustain when the tax burden for all citizens is decreasing. On the surface, it seems that, at some point, fairness and equity might no longer be the main requirement because the
"rich simply get richer and the poor get poorer." Indeed, the effect of "tax relief for the rich" seems to affect the well-being of less fortunate citizens adversely. In the face of these controversies, no one can estimate the extent of potential fallout that might result from such outcomes. As a voter, the reader is invited to contribute to this analysis and attempt to answer this dilemma.

The citizens are those that should ultimately decide what needs to be done in order to socially plan and redesign the wealth redistribution rules and norms. Taking advantage of this opportunity, it is instructive to perform an exercise related to the most appropriate choice of welfare policy, as shown in the "minimizing wealth-tax" column of Table 1.\(^5\) We estimated that tax relief for all citizens—despite minimizing the tax burden—is, in fact, fiscally safe, while also ensuring just and fair redistribution of wealth for all citizens.

The following discussion deserves, perhaps, some guidance, due to the assumptions made during the analysis. Before commenting on those, it is worth noting that the study presented here should be understood as purely normative—namely, "what ought to be" in economic or political matters, as opposed to "what is." Therefore, despite the fact that, in the preceding analysis, no actual situation was presented, our theoretical results rest on the assumptions delineated below.

First, we worked under the premise that politicians only made promises that can be fulfilled—fiscally save proposals. However, fiscal safety, taken separately, even when attempted in accordance with the rules and norms in force, could lead to unjust and unfair solutions. Indeed, taken at will, fiscal safety might be a profoundly mistaken idea of justice. In Table 1, we present the percentage of citizens below the poverty line, thus establishing the poverty rate.\(^6\) Driven at will, the official poverty rate, in accordance with the disagreement column in Table 1, could cause the poverty rate to decline below 0.41%, which wrongly appears to be the most just and fair.

Second, the wealth redistribution compensated for the inequalities in the income of citizens that were below the poverty line. Usually, similar parameters are in the national government competence. While taking into account increases in the cost of living, the official number of individuals living in poverty should be adjusted annually according to government guidelines. Although our key assumption was that the right-wing politicians inherited no more than an advisory authority, the rules and norms that govern the poverty line determination have been solely under the mandate of the left-wing politicians. This decision was made because, in the analysis, we deliberately emphasized the distinctions between stereotypical motivations of left-right wing politicians. In our view, welfare protection that is most likely to be just as fair should be addressed as an independent institute, or better yet, as an assembly of independent institutes or legal charity foundations. We believe that, in our study of organizational independence, welfare protection could be expected to yield efficient welfare policies. Thus, in determining an efficient policy on poverty, we concluded that left-wing politicians should be in a privileged position that allows them to prescribe the poverty line independently. Only when these guidelines of independence are applied, the value judgment based upon the data presented in Table 1 makes sense. Still, it should be noted that the characterization of whether setting up such a privilege was a positive or negative restriction requires additional investigation.

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5 Table 1 was created by numerical simulation carried out upon imaginary distribution of citizens' incomes.

6 Poverty rate determines the percent of anyone who lives with income less than official poverty line. The poverty line separates the rich (those with an income higher than the line), from the less fortunate (with an income below the line).
Next, we focused on the political power indicators \((\alpha, 1-\alpha)\), which highlight the amount of resources, skills and competence of left-right wing politicians. The fundamental factor in our analysis was the welfare protection of the society as a whole to justify and maintain welfare duties under the principle of how the state ought to act when trying to fulfill its welfare mission. When the decision made by the politicians is not in line with the wants of special interest groups, as previously pointed out, welfare protection could be a never-ending theme in political debates and election campaigns, and a source of significant political competition. A controversy of political interests might lead to violent upsets, providing the opportunity to develop policy in favor of these groups. According to the foregoing account, which requires considerable administrative efforts and fiscally unrealistic expenses—and previous observations pertaining to the independence of the welfare services—we believe that there is no reason to have sophisticated left-wing institutions. Thus, recognizing the vital role of the right-wing politicians, due to their central position to decide who will be purchasing and delivering public goods, in the interpretation of the parameter \(\alpha\), we believed that it was beneficial to impose a lower grade \(\alpha\) to the left-wing politicians, with a corresponding higher grade \(1-\alpha\), assigned to the right-wing politicians, \(\alpha < 1-\alpha\), \(0 < \alpha < 1\). Thus, it was reasonable to assume that left-wing politicians, with almost no extra effort, would demonstrate an ample degree of readiness to make efficient decisions. Herewith, in planning and regulating the size of the wealth-pie to suit a fiscally realistic welfare policy to settle and assist the state welfare mission, we tried to redesign the balance of political powers between the left-right wing politicians by adjusting the power indicator \(\alpha\) imposed on the left- and \(1-\alpha\) on the right-wing politicians. This enabled us, in order to benefit all citizens in society, in our view, to adjust the state rules and norms of the wealth redistribution, aligning it closer to the legal responsibilities and moral obligations of the citizens. We called the process of adjusting the power indicators \((\alpha, 1-\alpha)\) a political power design. Such a politically designed outcome was, as we supposed, worth the time and effort, even if the vision was a utopia.

The design of political power indicators \((\alpha, 1-\alpha)\) is a difficult and extremely time-consuming process. As prolonged political efforts might not be in the interest of anyone—citizens might not pursue it, even if the balance of political power can be ultimately reached. In particular, we supposed that electoral maneuvering of voters might put prolonged political efforts at risk of a premature collapse. It was deemed acceptable to assume presence of an implicit risk of voters defecting to the other side, which could interrupt negotiations ahead of the schedule. Thus, we brought the problem of likelihood of negotiations collapsing into focus. In our study that the failure of negotiations was extremely undesirable for both politicians, we hoped that this would be an incentive to move toward a solution faster. Alternatively, they would be more motivated to agree on terms of a contract, where both sides approach each other by making considerable concessions. In the view of welfare receipt of subsidies, a policy of higher tax rates might be the most favorable and just solution for minority. However, from the majority perspective, the minimum tax rate always lies first at peoples' wants. For the citizens who finance the subsidies, as we assumed in the analysis, the minimum tax rate provided a more just and fair redistribution of wealth. The minimum rate also provided an outcome \(\lambda\) in which the designed political power indicators \((\alpha, 1-\alpha)\) visualize the society's common denominator. Assuming, as we previously did, in accordance with the rules of the game, that outcome \(\lambda\) minimizing taxes could be politically designed—the outcome \(\lambda\) provided insight into what policy should entail.
Table 1, presenting all four assumptions, suggests several proposals for citizens to vote on. Note that, when voting for policy of equal left-right wing political power, the policy $\eta = 79.23$, is less just and less fair than the outcome $\lambda = 45.50$, where the minimum 26.52% of marginal tax rate is reached. Thus, only policy/outcome $\lambda$ on the poverty line (Figure 5) can be the only desirable political consent. Indeed, in the variety of rules in the game the left-right wing politicians play, when engaged in an interaction aimed at implementing equal/egalitarian policy $\eta$, the equal political power $\alpha = 0.5$ of the LWP was stronger than 0.21. However, consumers' goal can still be achieved by applying the weaker policy $\lambda = 45.50$ for the tax rate 26.52% < 28.21%, although the outcome of the weakened political power indicator $\alpha = 0.21$ is yet to be confirmed. Therefore, through a reduction of citizens' obligations—even with LWP's weakened political position—the LWP will be able to come to a desirable agreement with the RWP maintaining the most just and fair poverty line of wealth for all citizens.

In closing the discussion, we would like to point to a decision $\delta$ that corresponds to the breakdown of negotiations. Utopian society, planned according to the event of a breakdown, as shown in Table 1, seemingly ignores welfare protection because practically all citizens are considered rich by default. In other words, poverty does not exist. Given this utopian society, financing expenses almost entirely with respect to vital public/non-basic goods, the breakdown policy $\delta$, on equity condition, requires $-2.49$ public debt per capita. This, in turn, will require borrowing or money printing, e.g., promoting public spending through natural assets for refunding the debt. We, therefore, admit that, based on the lowest tax burden of 26.52%, a self-financing tax system has a better chance of being implemented.

8. Concluding remarks

This study has contributed important knowledge to the field. We followed the delivery of basic goods under the wealth redistribution rules and norms of low incomes compensations' terms. By negotiating the terms along the edge of the poverty line that treated all citizens equally, the politicians representing the opposing sides of the bargain decided how the basic and vital goods delivery should be financed. The expenses of the delivery pertaining to basic goods, as well as those associated with public (non-basic) vital goods, were separately estimated by transforming the expenses into functions of the poverty line. Based on the analysis, we concluded that an elevated poverty line, as a parameter, gave rise to inverse working incentives of benefits claimants, referred to as the hazard or h-factor effect. This resulted in unbalanced books accounting for the delivery of basic and non-basic goods to their respective destinations. For this reason, contra h-factor, the balance became crucial in resolving the political controversy between left-right wing politicians, as the two key actors in the bargaining game.

Given the controversial interests of the left-right wing politicians, and the need to resolve the welfare policy dilemma, both actors were willing to make concessions. The root of the controversy was that, in pursing their own political causes, the left-wing politicians struggled—in response to public wants—for the increase of basic goods, whereas the right-wing politicians advocated for meeting the needs for non-basic goods. Left-wing politicians gave credit to the tax system to guarantee a reasonably high living standard for benefit claimants. However, whatever public spending voters preferred, both politicians were aware of voters' electoral maneuvering, which could put the negotiations at risk of premature collapse. This threat was the only driving force in reaching the consensus. We argued that political arguments demanding higher taxes were weak, as overly costly proposals must subsidize the
excessive number of benefits claimants, which, in spite of the tax increase, could lead to diminished quality of the welfare services. In turn, the excessive number of claims could generate further requests for the additional financial support through tax channels. In order to satisfy those who bear additional costs, and who could only approve the requests on the terms of fiscally safe welfare policies, we reduced the scope of negotiations to the fiscally realistic domain of voters’ expectations.

In view of the above, a pretext for the analysis of the domain and the extent of bargain portfolio of two visionary politicians, denoted as $LWP$ and $RWP$, were established. The portfolio was supposed to account for politicians having non-conforming expectations. Instead of the wealth-pie division, such an account allowed for the inclusion a guide on how the eventual consensus ought to be analyzed and interpreted within the scope of negotiations $\left[\varepsilon_1, \varepsilon_2\right]$ at the contract curve. In this context, the left-right wing political power indicators, specified by the bargaining problem solution, were supposed to be politically designed in advance to tailor them in accordance with the citizens’ visions and ambitions.

It was initially deemed that, due to the uncertainty in the selection of the breakdown policy, we could only treat the left-right wing political power indicators as given exogenously. However, at least true in the valuable examples, a condition, where we can encode the indicators endogenously, was found and named "the pre-equity of breakdown."

References

Appendices

A1. Example and Illustration

We proceed with a specific allocation of the welfare policy encapsulating samples of income density distribution, parameterized by poverty line $\xi$, similar to an exponential function:

$$P(\sigma, \theta + h \cdot \xi) = \frac{1}{(\theta + h \cdot \xi) \cdot \Gamma(m)} \left( \frac{\sigma}{\theta + h \cdot \xi} \right)^{m-1} \cdot \exp\left( - \frac{\sigma}{\theta + h \cdot \xi} \right),$$

where $\theta = 61.9$, $m = 2.07$, and $h = -0.18$ are additional ex-ante parameters. More specifically, $\theta$ controls the wealth of citizens—a horizontal shift of samples; $m$ controls inequality—a vertical shift; $h$ is a hazard parameter; and $\Gamma(m)$ is an extension of $(m-1)!$ to real numbers.

The density function $P(\sigma, \theta + h \cdot \xi)$, depending on $\xi$, reflects the initial wealth redistribution through tax channels. Political decision $\xi > \xi'$ shifts the density distribution $P(\sigma, \theta + h \cdot \xi)$ of incomes horizontally toward the allocation $P(\sigma, \theta + h \cdot \xi')$ that favors less wealthy. When shifted, the distribution $P(\sigma, \theta)$ masks the h-factor, $h = 0$, of the benefit claimants. The rate of change $Hz(\xi) = h \cdot \dot{a}(\theta + h \cdot \xi) < 0$ of the policy $\xi$ quantifies a fiscally tolerable hazard ($h < 0$).

![Figure 3](image)

**Figure 3.** At the sample $P(\sigma, \theta + h \cdot \xi)$ of the income density distribution, $\mu$ solves the equation $\int_{0}^{x} P(\sigma, \theta + h \cdot x) \, d\sigma = 0.5$ for $x$; $\mu = 82.30$.

---

7 The sample $\xi = \frac{1}{2} \mu$ (median income $= \mu$) can be presented as Lorenz Curve, where citizens below an income $151.48$, i.e., 75% of the population, have $51.11\%$ of a total cumulative income, while the remaining 25%, with incomes at or above $151.48$, have $48.89\%$. Gini Coefficient equals 0.37. Horizontal shifts do not affect the Gini coefficient, while the vertical ones do. A more detailed example is available upon request.
A2. Simulation results

In order to perform simulations, the expressions for average $B(\xi)$ of expenses on the subsidies and average taxable income—the wealth amount $W(\xi)$—can incorporate income density distribution $P(\sigma, \theta + h \cdot \xi)$ in a more realistic but general form:

$$B(\xi) = r \cdot \int_{0}^{\xi} (\xi - \sigma) \cdot P(\sigma, \theta + h \cdot \xi) \, d\sigma; \quad r \cdot (\xi - \sigma)$$ is the LI-subsidy, $0 < r < 1;$$

$$W(\xi) = \int_{0}^{\xi} (\sigma + r \cdot (\xi - \sigma) - \phi) \cdot P(\sigma, \theta + h \cdot \xi) \, d\sigma + \int_{\xi}^{\infty} (\sigma - \phi) \cdot P(\sigma, \theta + h \cdot \xi) \, d\sigma.$$

In the left-right political bargaining, the choice of $\xi$, in general, is also an issue of the average income $a(\theta + h \cdot \xi)$ maintenance to uphold $a(\theta + h \cdot \xi) > W(\xi)$ within the “striking” distance from $W(\xi)$, which can be ensured through proper choice of the personal allowance constant $\phi > 0$, where $\phi$ identifies a flat tax bracket $[\phi, \infty)$. The average $a(\theta + h \cdot \xi)$ of income $\sigma$ over the density sample $P(\sigma, \theta + h \cdot \xi)$ equals $\int_{0}^{\infty} \sigma \cdot P(\sigma, \theta + h \cdot \xi) \, d\sigma$.

The taxation of the total income $\sigma + r \cdot (\xi - \sigma)$ of the needy is in compliance with the rules and norms in force; the $h$-factor reveals the inverse working incentives—namely, the feedback of the welfare recipients. These rules and norms are in line with our amendment to Friedman [8] proposals.

At this point, it is useful to verify that a disagreement policy $\delta$ under the primacy of equity principle of breakdown might be an outcome of the game. There is no reason why the equation $(\delta - \phi) \cdot (B(\delta) + d_2) - (\delta - d_1) \cdot W(\delta) = 0$, in accordance with the observation 3, should have a solution in general. However, for the income density $P(\sigma, \theta + h \cdot \xi)$ (see above), a solution can be found. Given payoffs $(u, g)$ at the endpoints $(u_1 = 6.44, g_1 = 47.18)$, $(u_2 = 89.26, g_2 = -2.49)$ of the scope of negotiations—within the interval $[\xi_1 = 8.00, \xi_2 = 144.54]$—one can discover that the pair $d = (d_1 = u_1, d_2 = g_2) = (6.44, -2.49)$, $u_1 < u_2$, $g_1 > g_2$ consolidates an equity for breakdown policy $\delta = 6.39 \not\in [\xi_1, \xi_2]$; wealth $W^* = 120.46$ and tax $\tau^* = -2.06\%$.

It should not be surprising that the amounts of public goods and tax rates may be negative. Ensuring this outcome of the game, the interpretation suggests that the simulated breakdown demonstrates a specific payoff deficit on public goods when it is impossible to cover the costs through taxes. In such a scenario, as we have pointed out earlier, when discussing the breakdown, the solution is resorting to an external loan, money printing, or use of natural resources, if the latter are available.
The magnitude and dimension of poverty proposals to be debated or implemented, as "outcomes of the left-right political bargaining," are given in the following table.

Recall already known proposals for incomes $\eta, \lambda_1, \lambda, \lambda_2, \delta$ —whereby $\delta$ is outside of the scope of negotiations, $\delta \notin [\xi_1, \xi_2]$; and the poverty proposal $\frac{1}{2}\mu$, as follows:

- $\eta$ the policy on poverty with equal left-right wing political power; the left-right wing political organizations are in symmetrical positions or in equal roles;
- $\lambda_1$ the outcome of the alternating-offers' game—what the right-wing politicians accept;
- $\lambda$ the policy on poverty minimizing wealth-tax;
- $\frac{1}{2}\mu$ 50% of the median income, indicating that half of the population earn income above $\mu$, with the income of the remaining half below $\mu$;
- $\lambda_2$ the outcome of the alternating-offers' game—what the left-wing politicians accept;
- $\delta$ the least desirable outcome, resulting in the breakdown policy or disagreement, which naturalizes the risk of negotiations' premature collapse, caused, for instance, by mutual traps.

<table>
<thead>
<tr>
<th>Table 1. Numerical simulation behind the left-right wing political power design; LWP—left-wing politicians, RWP—right-wing politicians</th>
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</thead>
<tbody>
<tr>
<td><strong>Obtained by means of income density distribution (Figure 3); personal allowance $\phi = 4.03; \theta = 61.9; h = -0.18; m = 2.07; r = \frac{1}{2}; q = 5%$</strong></td>
</tr>
<tr>
<td>Policy of equal—politically symmetric power</td>
</tr>
<tr>
<td>$\eta$</td>
</tr>
<tr>
<td>Poverty line; welfare policy</td>
</tr>
<tr>
<td>Poverty rate: percentage of citizens below the poverty line</td>
</tr>
<tr>
<td>Political power of left-wing politicians</td>
</tr>
<tr>
<td>LI netto; the after-tax residue of $\xi$</td>
</tr>
<tr>
<td>Account for public goods expenses</td>
</tr>
<tr>
<td>Account for LI subsidies transfers</td>
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<tr>
<td>Account for public spending, the size of the wealth-pie</td>
</tr>
<tr>
<td>Average taxable income—the wealth amount</td>
</tr>
<tr>
<td>Wealth-tax, marginal tax rate</td>
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</table>
Figure 4. The wants of left-wing politicians conflicting with right-wing political objective are depicted on the vertical and horizontal axes, respectively.

The graph shows the contract curve sloping from $\xi_2$ toward $\xi_1$, projected on the surface of basic goods contra vital goods—the projection of efficient poverty lines $\xi \in [\xi_1, \xi_2]$, resolving the contract constraint (5).

Figure 5. The graph depicts two different motions for a vote: for the higher tax $\tau = 29.01\%$, marked by the horizontal line, and the lowest tax $\tau = 26.52\%$, marked by the vertical dash. Indicated by $\rightarrow$, at cross points of the contract curve with the horizontal line, we observe controversial expectations of voters. The shares of lower basic but higher public goods are shown on the left, while this payoffs reverses towards the right side of the graph, as the shares of basic goods increase while those for public goods decrease. Thus, the higher tax $\tau = 29.01\%$ cannot stand for a political consent, Observation 5.
A3. Verification

Proof of observation 1. Let us now assume an inverse scenario, whereby \( u > u' = \pi(\xi, \tau(\xi, x)) \). Here, the left-wing politicians—LWP—aim to improve the poverty line residue \( u' \), i.e., an after-tax residue of a marginal citizen \( \sigma = \xi \) with income equal to the poverty line \( \xi \). By initiating a new rule for policy \( \xi' > \xi \), the LWP attempt to implement \( u > u' \). Because of the inequalities \( u \geq \pi(\sigma, \tau(\xi, x)) > u' \), for some highly pragmatic benefit claimants \( \sigma \), it becomes apparent that they can be better off by claiming subsidies. Consequently, actions of these claimants will increase the expenditure \( B(\xi') > B(\xi) \) on the subsidies and shift the balance of books \( B(\xi) = x \cdot \tau(\xi, x) \cdot W(\xi) \) to deficit \( B(\xi') > x \cdot \tau(\xi, x) \cdot W(\xi) \). The balance was valid in the past, when \( \tau(\xi, x) = \frac{B(\xi)}{x \cdot W(\xi)} \). Thus, the only option that would ensure that the balance in maintained, as the LWP must stay committed to \( x \), is to adjust \( \tau(\xi, x) \) to \( \tau(\xi, \xi', x) = \frac{B(\xi')}{x \cdot W(\xi)} > \tau(\xi, x) \) — \( x \) was fixed by the agreement. Otherwise, keeping the old policy \( \xi \) intact, the LWP could—through a decrease in \( x \)—violate the commitment \( x \).

However, as they cannot directly change \( x \), they resort to reducing the deficit by tax increase. If \( u > \pi(\xi', \tau(\xi', \xi', x)) \), the LWP must continue with the tax adjustment policy by \( \tau(\xi', \xi'', x) > \tau(\xi, \xi', x) \); however, adjusting now upon the welfare policy \( \xi' \) and proposing \( \xi'' > \xi' \), the new deficit becomes \( B(\xi'') > x \cdot \tau(\xi, \xi', x) \cdot W(\xi') \). These improvements \( u > u'' > u' \) initiate a sequence of poverty policies \( (... \xi'' > \xi' > \xi,...) \) and after-tax residues \( (... u > u'' > u', ... \) of marginal citizens. Thus, the conditions \( u = u'' \) and \( \xi = \xi'' \) can never be met, as this would contradict the assumption that the equation \( u = \pi(\xi, \tau(\xi, x)) \) cannot be resolved for \( \xi \). For this reason, the sequence \( ..., \xi'' > \xi', ... \) is infinite. ■

The chain of reasoning regarding \( u' > u \) is similar to above and is presented as a set of instructions. It should first be noted that, at low values \( u' > u'' > u \), even when taxes are low, there would always be a surplus to finance the LI-benefits and subsidies. However, the surplus masks a contradiction, since it is clear that, at low values of the after-tax residue parameter \( u \), benefits financing can always be balanced.

<table>
<thead>
<tr>
<th>Replace</th>
<th>to implement an improved</th>
<th>by</th>
<th>to make a decline in</th>
</tr>
</thead>
<tbody>
<tr>
<td>– better off</td>
<td></td>
<td>– worse off</td>
<td></td>
</tr>
<tr>
<td>– improve</td>
<td></td>
<td>– decline</td>
<td>deterioration</td>
</tr>
<tr>
<td>– improvement</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>– to claim for subsidies</td>
<td></td>
<td>– that subsidies have been revoked</td>
<td></td>
</tr>
<tr>
<td>– deficit</td>
<td></td>
<td>– surplus</td>
<td></td>
</tr>
<tr>
<td>– ( \geq, &gt; )</td>
<td></td>
<td>– ( \leq, &lt; )</td>
<td></td>
</tr>
</tbody>
</table>

Transpose: an increase with a decrease
In what follows, we investigate the payoffs \( \langle u, g \rangle \in S_b \) of the left-right wing politicians. The consensus occurs at outcomes \( \phi, \xi \Rightarrow z, x, \alpha, \tau, \langle u, g \rangle \) under the constraint that the variation of policy \( \xi \) does not improve the position of the left-wing politicians; rather, the policy emerges as the point on the contract curve \( S_b = u(g) \) as fiscally idempotent outcome.

For fiscally idempotent outcomes, the arguments of after-tax residue \( u \), share \( x \), policy \( \xi \), and tax rate \( \tau \) depend on each other. The share \( x = x^o \), if settled as eventual agreement, redirects the residue \( u = \pi(\xi, \tau(x^o)) \) to become a function \( u = u(\xi, x^o) \). Thus, the peak policy \( u \) with regard to the best welfare policy can be expressed as

\[
\xi^o = \arg \max_{\xi} u(\xi, x^o)
\]  

(A3.1)

**Lemma.** Let us assume that left-wing politicians do not shift from the share \( x = x^o \) and that the volatility-constraint (4) solves for two different policies \( \xi_1 < \xi_2 \). Let the tax sacrifice 
\[
t(\xi, x^o) = \tau(\xi, x^o) \cdot (\xi - \phi)
\]
be a differentiable function of \( \xi \) progressively increasing with \( \xi \) within the closed interval \( [\xi_1, \xi_2] \)—namely, the derivatives

\[
\frac{\partial}{\partial \xi} t(\xi, x^o) \Big|_{\xi=\xi_1} > 0, \quad \frac{\partial}{\partial \xi} t(\xi, x^o) \Big|_{\xi=\xi_2} < 0 \quad \text{and} \quad \frac{\partial^2}{\partial \xi^2} t(\xi, x^o) > 0.
\]

In such situation, the poverty line residue \( u(\xi, x^o) = \xi - t(\xi, x^o) \) is a single \( \Rightarrow \)-peaked function of \( \xi \).

**Corollary.** There exists a unique interior policy \( \xi^o \) maximizing \( u \) at

\[
\frac{\partial}{\partial \xi} u(\xi, x^o) \Big|_{\xi=\xi^o} = 0.
\]

Provided that the conditions of the lemma are fulfilled, the discussion that follows concerns the necessary and sufficient conditions for the fiscally idempotent policy \( \xi \) to occur at the contract curve.

**Observation 2.** Let us assume that the volatility-constraint (4) is differentiable from its arguments. The after-tax residue \( u = u(\xi, x^o) \) is differentiable and single peaked with respect to the policy \( \xi \) within some closed interval \( [\xi_1, \xi_2] \). For a fiscally idempotent outcome \( \phi, \xi^o \Rightarrow z^o, x^o, \alpha, \tau^o, \langle u^o, g^o \rangle \) to occur on the contract curve \( S_b = u(g) \), it is necessary and sufficient that the policy \( \xi^o \) solves the set of equations:

\[
(i) \quad \frac{\partial}{\partial \xi} L(\xi, x^o, u^o) \Big|_{\xi=\xi^o} = 0, \quad \text{where} \quad u^o = u(\xi^o, x^o) \quad \text{provided that}
\]

\[
(ii) \quad \frac{\partial}{\partial u} L(\xi^o, x^o, u) \Big|_{u=u^o} \neq 0.
\]
Proof

**Necessity.** Let the fiscally idempotent outcome \( \phi, \xi^o \Rightarrow z^o, x^o, \alpha, \tau^o, \langle u^o, g^o \rangle \) on the contract curve \( S_b = u(g) \) maximize (A3.1) at \( u^o = u(\xi^o, \tau(\xi^o, x^o)) \). Varying \( \xi \) in the vicinity of \( \xi^o \) of the outcome \( \phi, \xi^o \Rightarrow z^o, x^o, \alpha, \tau^o, \langle u^o, g^o \rangle \) and substituting \( u = u(\xi, \tau(\xi, x^o)) \) into the volatility-constraint (4), we obtain an identity \( L(\xi^o, x^o, \pi(\xi, \tau(\xi^o, x^o))) \equiv 0 \). Within the proximity of \( (\xi^o, u^o) \), the following equation holds for arguments \( \xi, u \):

\[
\frac{\partial}{\partial \xi} L(\xi, x^o, u^o) + \frac{\partial}{\partial u} L(\xi^o, x^o, u) \cdot \frac{\partial}{\partial \xi} \pi(\xi, \tau(\xi, x^o)) = 0,
\]

(A3.2)

from which we deduce the necessity statement for \( \xi = \xi^o \) and \( u = u^o \).

**Sufficiency.** Suppose the condition (ii) holds. Let (i) solve for \( \xi^o \) at the fiscally idempotent outcome \( \phi, \xi^o \Rightarrow z^o, x^o, \alpha, \tau^o, \langle u^o, g^o \rangle \). Combining (i) and (A3.2), we conclude that

\[
\left. \frac{\partial}{\partial \xi} \pi(\xi, \tau(\xi, x^o)) \right|_{\xi = \xi^o} = 0.
\]

The sufficiency clause (A3.1) holds, since \( u = u(\xi, x^o) \) is a convex function of \( \xi \).

Proof of observation 3. The clause is correct, provided that there exists a fiscally idempotent policy \( \delta \) for the implementation of the pair \( \langle d_1, d_2 \rangle \). In order to identify such a policy, we first replace the variable \( g \) with \( d_2 \) in the expression for the constraint (1). Next, we extract the expression for \( \tau = \frac{B(\delta) + d_2}{W(\delta)} \) from (1) and substitute it into \((1 - \tau) \ldots \) of the constraint (3), where \( u \) should be replaced by \( d_1 \) in advance. By simplifying, we arrive at the statement of the observation.

Sketch of the proof (observation 5). Looking at the tax rate \( \tau > \tau_{\min} \), for any outcome \( \ldots, \tau, \langle u, g \rangle \in S_b \), one may indeed prefer a counter outcome as a motion \( \ldots, \tau, \langle u', g' \rangle \), which outlines \( \ldots, \tau, \langle u' > u, g' < g \rangle \) or \( \ldots, \tau, \langle u' < u, g' > g \rangle \). As the contract curve \( S_b = u(g) \) is a curve of efficient preferences \( \langle u, g \rangle \) guaranteeing the poverty line residue \( u(g) \), someone could put a motion \( u' > u^o \) or \( g' > g^o \) against an outcome \( \ldots, \tau > \tau_{\min}, \langle u^o, g^o \rangle \). We argue that, in order to fulfill the expectations and requests of citizens' majority, it is necessary to carry out political consent using the proposal \( \ldots, \tau_{\min} = \tau(\lambda), \langle u^o = u(\lambda), g^o = g(\lambda) \rangle \).
A4. Mathematical derivation

\[ \tau \cdot W(\xi) = B(\xi) + g \]

**Delivery-constraint**: the size of the welfare-pie, i.e., the average tax returns amount equals the average monetary value per capita of primary goods plus average of non-primary goods \( g \).

\[ B(\xi) = x \cdot \tau \cdot W(\xi) \]

**Budget-constraint** imposed on the subsidies finance in accordance with the share \( x \) of the wealth-pie—the tax-revenue.

\[ u = (1 - \tau) \cdot (\xi - \phi) + \phi \]

**Stability constraint** that determines fiscally idempotent policy \( \xi \).

\[ u = \xi - \tau \cdot (\xi - \phi) \]

**After-tax residue constraint**: an alternative form of stability constraint, where \( u \) is after-tax position of a marginal citizen with income \( \sigma = \xi \). The left-wing political wants.

Replacing \( \tau = \frac{B(\xi)}{x \cdot W(\xi)} \) from the budget-constraint into the stability constraint, we obtain the volatility-constraint (4) as stated:

\[ L(\xi, x, u) = (\xi - \phi) \cdot B(\xi) - x \cdot (\xi - u) \cdot W(\xi) = 0 \]

that amalgamates budget-constraint and after-tax residue. Contract curve (5) is thus given by:

\[ D(\xi, x, u) = L'_\xi(\xi, x, u) = \left[ (\xi - \phi) \cdot B(\xi) - x \cdot (\xi - u) \cdot W(\xi) \right] = 0; \]

\[ L'_\xi(\xi, x, u) = B(\xi) + (\xi - \phi) \cdot B'(\xi) - x \cdot W(\xi) - x \cdot (\xi - u) \cdot W'(\xi) = 0. \]

The last expression may be rewritten as:

\[ D(\xi, x, u) = B(\xi) + (\xi - \phi) \cdot B'(\xi) - x \cdot [W(\xi) + (\xi - u) \cdot W'(\xi)] = 0. \]

Extracting \( x = \frac{(\xi - \phi) \cdot B(\xi)}{(\xi - u) \cdot W(\xi)} \) from the volatility-constraint (4), we can substitute variable \( x \) into the rewritten expression for \( D(\xi, x, u) \). The substitution results in the following expressions:

\[ B(\xi) + (\xi - \phi) \cdot B'(\xi) = \frac{(\xi - \phi) \cdot B(\xi)}{(\xi - u) \cdot W(\xi)} \cdot [W(\xi) + (\xi - u) \cdot W'(\xi)] = 0, \]

or

\[ \frac{[B(\xi) + (\xi - \phi) \cdot B'(\xi)] \cdot (\xi - u) \cdot W(\xi) - (\xi - \phi) \cdot B(\xi) \cdot [W(\xi) + (\xi - u) \cdot W'(\xi)]}{(\xi - u) \cdot W(\xi)} = 0. \]

Provided that \( (\xi - u) > 0 \) and \( W(\xi) > 0 \), we can conclude that the following is true:
This allows writing the sub-expression \( (\xi - u) \) in the form:

\[
(\xi - u) - \phi \cdot B(\xi) \cdot W(\xi) = 0.
\]

As a consequence of presenting the sub-expression \( (\xi - u) \) in the form given above:

\[
\xi - u = \frac{(\xi - \phi) \cdot B(\xi) \cdot W(\xi)}{B(\xi) + (\xi - \phi) \cdot B'(\xi)} \cdot \frac{B(\xi) \cdot W(\xi)}{B(\xi) + (\xi - \phi) \cdot B'(\xi)} \cdot \frac{B(\xi) \cdot W(\xi)}{B(\xi) + (\xi - \phi) \cdot B'(\xi)}.
\]

We can now substitute the tax rate \( \tau \) from the delivery-constraint into the after-tax residue constraint. The result will be \( u = \xi - \frac{B(\xi) + g}{W(\xi)} \cdot (\xi - \phi) \). After replacing the result into the observed \( u \) expression, we obtain:

\[
\xi - \frac{B(\xi) + g}{W(\xi)} \cdot (\xi - \phi) = \xi - \frac{(\xi - \phi) \cdot B(\xi) \cdot W(\xi)}{B(\xi) + (\xi - \phi) \cdot B'(\xi)} \cdot \frac{B(\xi) \cdot W(\xi)}{B(\xi) + (\xi - \phi) \cdot B'(\xi)} \cdot \frac{B(\xi) \cdot W(\xi)}{B(\xi) + (\xi - \phi) \cdot B'(\xi)};
\]

\[
\frac{B(\xi) + g}{W(\xi)} \cdot (\xi - \phi) = \frac{(\xi - \phi) \cdot B(\xi) \cdot W(\xi)}{B(\xi) + (\xi - \phi) \cdot B'(\xi)} \cdot \frac{B(\xi) \cdot W(\xi)}{B(\xi) + (\xi - \phi) \cdot B'(\xi)} \cdot \frac{B(\xi) \cdot W(\xi)}{B(\xi) + (\xi - \phi) \cdot B'(\xi)};
\]

\[
[B(\xi) + g] \cdot (\xi - \phi) = \frac{(\xi - \phi) \cdot B(\xi) \cdot W(\xi) \cdot W(\xi)}{B(\xi) + (\xi - \phi) \cdot B'(\xi)} \cdot \frac{B(\xi) \cdot W(\xi) \cdot W(\xi)}{B(\xi) + (\xi - \phi) \cdot B'(\xi)} \cdot \frac{B(\xi) \cdot W(\xi) \cdot W(\xi)}{B(\xi) + (\xi - \phi) \cdot B'(\xi)}.
\]

\[
B(\xi) + g = \frac{B(\xi) \cdot W(\xi) \cdot W(\xi)}{B(\xi) + (\xi - \phi) \cdot B'(\xi)} \cdot \frac{B(\xi) \cdot W(\xi) \cdot W(\xi)}{B(\xi) + (\xi - \phi) \cdot B'(\xi)} \cdot \frac{B(\xi) \cdot W(\xi) \cdot W(\xi)}{B(\xi) + (\xi - \phi) \cdot B'(\xi)};
\]

\[
g = \frac{B(\xi) \cdot W(\xi) \cdot W(\xi)}{B(\xi) + (\xi - \phi) \cdot B'(\xi)} \cdot \frac{B(\xi) \cdot W(\xi) \cdot W(\xi)}{B(\xi) + (\xi - \phi) \cdot B'(\xi)} \cdot \frac{B(\xi) \cdot W(\xi) \cdot W(\xi)}{B(\xi) + (\xi - \phi) \cdot B'(\xi)} - B(\xi).
\]

We can now impose the denominator in the last expression for \( g \) on sub-expression for \( (\xi - \phi) \), which can be written as:
\[ [B(\xi) + (\xi - \phi) \cdot B'(\xi)] \cdot W(\xi) - (\xi - \phi) \cdot B(\xi) \cdot W'(\xi) = \]
\[ = B(\xi) \cdot W(\xi) + (\xi - \phi) \cdot [B'(\xi) \cdot W(\xi) - B(\xi) \cdot W'(\xi)]. \]

Continuing with the expression for \( g(\xi) \), we can replace the denominator transformed above:

\[
g = \frac{B(\xi) \cdot W(\xi) \cdot W(\xi)}{B(\xi) \cdot W(\xi) + (\xi - \phi) \cdot [B'(\xi) \cdot W(\xi) - B(\xi) \cdot W'(\xi)]} - B(\xi); \]

\[
g = \frac{B(\xi) \cdot W(\xi) \cdot W(\xi) - B(\xi) \cdot (B(\xi) \cdot W(\xi) + (\xi - \phi) \cdot [B'(\xi) \cdot W(\xi) - B(\xi) \cdot W'(\xi)])}{B(\xi) \cdot W(\xi) + (\xi - \phi) \cdot [B'(\xi) \cdot W(\xi) - B(\xi) \cdot W'(\xi)]}, \]

Now, both the nominator and the dominator can be divided by \( B(\xi) \cdot W(\xi) \), yielding:

\[
g = \frac{W(\xi) - B(\xi) \cdot \left\{ \frac{B(\xi) \cdot W(\xi) + (\xi - \phi) \cdot [B'(\xi) \cdot W(\xi) - B(\xi) \cdot W'(\xi)]}{B(\xi) \cdot W(\xi)} \right\}}{B(\xi) \cdot W(\xi) + (\xi - \phi) \cdot [B'(\xi) \cdot W(\xi) - B(\xi) \cdot W'(\xi)]}. \]

Let us define \( v(\xi) = 1 + (\xi - \phi) \cdot \left\{ \frac{B'(\xi)}{B(\xi)} - \frac{W'(\xi)}{W(\xi)} \right\} \), as this allows us to evaluate the expression for the right-wing political objective on public but vital goods as:

\[
g(\xi) = \frac{W(\xi) - B(\xi) \cdot v(\xi)}{v(\xi)} = \frac{W(\xi)}{v(\xi)} - B(\xi). \]

In accordance with the delivery-constraint, the size of the wealth-pie \( \tau(\xi) \cdot W(\xi) \) equals \( B(\xi) + g(\xi) \). Consequently, the tax rate is given by:

\[
\tau(\xi) = \frac{B(\xi) + g(\xi)}{W(\xi)} = \frac{B(\xi) + \left( \frac{W(\xi)}{v(\xi)} - B(\xi) \right)}{W(\xi)} = \frac{1}{v(\xi)}.
\]

Replacing the \( \tau = \frac{1}{v(\xi)} \) in the after tax residue \( u = \xi - \tau \cdot (\xi - \phi) \), we can finally evaluate the expression for the left-wing political wants on basic goods as:

\[
u(\xi) = \xi - \frac{(\xi - \phi)}{v(\xi)}.
\]