Equilibrium in a Retail Chain with Transaction Costs

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Abstract

The paper addressed a situation of how a retail chain consisting of suppliers, agents, and distributors transformed while the costs of transactions increased. When the costs increased, the orders and deliveries between relevant interest groups resulted in the formation of the most costs' tolerant retail chain. The participants of the most tolerant chain remained in equilibrium under condition that in any transaction the gain of trade exceeded the transaction cost. Making to buy and sale decisions, the participants of the chain supposed to follow the rules and norms of what the author called a monotonic game.

Keywords: suppliers, distributors, monotonic game, retail chain

Businessmen in deciding on their ways of doing business and on what to produce have to take into account transaction costs. If the cost of making an exchange are greater than the gains which that exchange would bring, that exchange would not take place and the greater production that would flow from specialization would not be realized. In this way transaction costs affect not only contractual arrangements, but also what goods and services are produced. Ronald H. Coase, “The Institutional Structure of Production,” Ménard, C., and M. M. Shirley (eds.) [2005], Handbook of New Institutional Economics, Spriner: Dordrecht, Berlin, Heidelberg, New York. XIII. 884pp., p.35, ISBN 1-4020-2687-0.

1. Introduction

All, perhaps, know that prices on commodity markets sometimes continue to rise unabated on the back of an anticipated shortage in the global raw materials availability and sharp volatility in the commodity future markets and terminal prices on fears of an immediate shortage of materials in the short term. Along with the significant increase in commodity prices, on one hand, the transaction costs increase on inputs like petroleum, electricity, etc. On the other, while currency of exchange rates also moving adversely, the situation becomes uncertain. As an example, one may point at recent market price increase of coffee raw materials, which did not have immediate consequences for some known positions, while the distributors 1 of a retail chain, however, demonstrate readiness to make loosing transactions. With this in mind, distributors are trying to hold prices constant. However, it is also understandable that it would be impossible for the distributor to make frequent price changes again and again. Given the current context, they will have no other option but to seek price increase for distributed commodities with an immediate effect.

Uncertainties in market prices of commodities always lead to an increase of transaction costs. Transaction costs increase once again leads to additional uncertainties, and the distributors in the retail chain end up in a dead circle of price increase, which may result that the bilateral trade does not take place, and the market old supply and demand structure to be replaced with a new. In the environment of constant price increase, the orders and deliveries do

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1 A group of retail outlets owned by one firm and spread nationwide or worldwide.
not match any more for a given supply and demand structure. In such situations, individual participants in the retail chain are still assumed to act rationally finding a new ways of making business with the object of maximizing the profit by trying to restructure the chain. Worth to note that New Institutional Economics gives an explanation for transactions as mediated through the market in two directions: the vertical integration, Joskow [2, 2005], where the market structure is mostly a vertical chain of semi-product components, and the horizontal chain of services and products outsourced by companies if needed to produce the end product.

This paper addresses the above situation in question by setting up a retail chain game of the participants in the chain grounding on supposition that orders and deliveries be met with uncertainty of transaction costs. In so doing, the paper attempts to develop a numerical description of the supply and demand structure for the deliveries of commodities in the retail chain. The allegedly rational behavior of a participant is not always such, because the participants on purpose may attempt to enter but irrationally into certain losing transactions in hope to offset the negative effect of the former. Given this irrational situation the prices will increase additionally upon already profitable transactions. Numerical analysis of irrational situations reveals, however, that in case the participants will try to avoid all losing transactions, their behavior is once again becoming rational and in such situations the participants of the retail chain will end up in the Nash equilibrium [8, 1953].

To our knowledge (or lack of that), the retail chain formation, or in mundane terms the restructuring process of the retail chain is rather complicated mathematical problem, which do not have satisfactory solutions. However, in recent years it has become clear that a mathematical structure known as antimatroid is well suited for such type a retail chain formation process, c.f., Algaba, et al. [1, 2004]. Antimatroid is a collection of potential interests groups—subsets of participants, i.e. those who make decisions to buy and sale in bilateral trade transactions. That is to say, within antimatroid one will always find a path of transactions connecting members of the retail chain—if the latter forms of course—with each other by mutual business interests inside groups/coalitions belonging to antimatroid and making the exchange as participants of a characteristic retail chain.

We step up beyond convention of the theory of coalition games that the solution mandatory has to be a core, and take the retail chain formation process in terms of so-called defining sequence of transactions, Mullat [6, 1979]. The sequence facilitates the retail chain formation as a transformation process of nested sets of bilateral transactions, which ends at its last and highest costs' threshold—the most tolerant retail chain towards costs—a kernel. Hereby, the kernel operates as a retail chain of participants capable to cover the highest transaction costs in case of uncertainty. In our case, the defining sequence of transactions produces the elements of an antimatroid—some interest groups, c.f., Levit and Kempner, [5, 2001]; see also Korte et al., [4, 1991]. The defining sequence on antimatroid, in particular, follows the Greedy heuristic procedure of Shapley's value, but in inverse order, c.f., Rapoport [10, 1985].
Bearing all this in mind, the suggested framework allows performing a series of computer simulations. First, to determine the possible response of the retail chain participants, to different supply and demand structures. Second, to identify the participants, where the executive efforts might be applied to prevent unpredictable actions that may misbalance the equilibrium in the retail chain. With this object, we used a model to assemble an “elasticity” measure for the choice of customers; this measure is represented by transaction costs’ interval, for which the retail chain remains in equilibrium.

The rest of this paper is structured as follows. The next section sets up the basic concepts intending to bring at the surface the calculus of utilities of participants in the retail chain. It is a preliminary step necessary to move forward to the Section 3, where the general model of participants of the chain is described. In Section 4, which is main part of the paper, the retail chain game of customers addresses the process of the chain formation in details. Here the monotonic property of utilities plays its major role. A summary of the results ends the study.

2. Description of a retail chain: the simple form

To consider the simplest case of commodities distribution in a retail chain might be instructive. This elementary model is used at current stage solely as a convenient means of simplifying the presentation.

The distribution of commodities in the retail chain is characterized by sales figures that may be expressed as one of the following three alternative numbers: a) a demand $\eta$ which is disclosed to the particular participant either externally or by other participant in the chain; b) a capable supply $\xi$ calculated at the cost of all commodities produced by the participant for delivery outside the chain or to the other participants; c) actual sales $\gamma$ calculated at the prices actually paid by the customers for the delivered commodities.

An order is thus defined as a certain quantity of a particular commodity ordered by one of the participant’s from another participant in the retail chain; a delivery is similarly defined as a certain quantity of a commodity delivered by one of the participant’s to another participant in the chain. We assume that the chain includes suppliers who are only capable of making deliveries – the produces; participants, who both issue orders and make deliveries – the agents; and the distributors, who only order commodities from other participants. 1

In what follows we consider the retail chain of orders and deliveries for the case like “pipeline” distribution without “closed circuits.” Therefor, we can always identify a unique direction of “retail chain” of orders from the distributors to the produces via agents and a “retail chain” of deliveries in the reverse direction.

1 In subsequent sections, the distributors also act as suppliers to external customers.
Let us consider in more detail this particular retail chain of orders and deliveries of commodities. The direction of the chain of orders (deliveries) is defined by assigning serial numbers – the indexes 1, 2 and 3 – to the producer, to the agent, and to the distributor, respectively. The producer and the agent act as suppliers, the agent and the distributor act as customers. The agent thus has the dual role of a supplier and a customer, whereas the producer only acts as a supplier and the distributor only acts as a customer.

The chain of orders to the produces from the customers is characterized by two numbers \( \eta_{23} \) and \( \eta_{12} \). The number \( \eta_{wj} \) (\( w = 1, 2; j = 2, 3 \)) is the demand \( \eta_{wj} \) disclosed by the customer \( j \) to the supplier \( w \). We assume that sales are equal to deliveries. Two numbers \( \xi_{12} \) and \( \xi_{23} \), which are interpreted as the corresponding capable sales similarly characterize the chain of deliveries to the distributor.

Suppose that the demand of the distributor to the external customers is fixed by \( d \) bank notes. The capable sales of the producer are \( s \) bank notes. In other words, \( d \) is the estimated amount of orders from the external customers and it plays the same role as the number \( \eta \) for the customers in the retail chain. Similarly, \( s \) is the intrastate amount of estimated deliveries by the producer, and it has the same role as \( \xi \) for the customers.

Let us now consider the exact situation in a chain. To make deliveries at a demand amount of \( d \) bank notes, the distributor have to place orders with the agent in the amount of \( \eta_{23} = \nu_{23} \cdot d \) bank notes, where \( \nu_{23} \) are the distributor’s cost of commodities sold (the cost per 1 bank note of sales). The agent, having received an order from the distributor, will in turn place an order with the supplier in the amount \( \nu_{12} \cdot \eta_{23} \), where \( \nu_{12} \) is the agent’s cost per 1 bank note of sales. On the other hand, the estimated sales of the producer are \( \xi_{12} \) bank notes, \( \xi_{12} = s \). Assuming that all the transactions between the suppliers and the customers in the retail chain are materialized in amounts not less than those indicated in the purchase orders, the actual sales of the producer to the agent are given by \( \gamma'_{12} = \min\{\xi_{12}, \eta_{12}\} \).

Now, since the agent paid the producer \( \gamma'_{12} \) for the commodities ordered, the agent’s revenue is \( \xi_{23} = \gamma'_{12} / \nu_{12} \), where clearly \( \xi_{23} \geq \gamma'_{12} \). The difference between the revenue \( \xi_{23} \) and the costs \( \gamma'_{12} \) is defined as \( \pi_{12} = \gamma'_{12} \cdot (1 - \nu_{12}) / \nu_{12} \).

From the same considerations, \( \gamma'_{23} = \min\{\xi_{23}, \eta_{23}\} \) give the actual sales of the agent to the distributor. We similarly define the difference \( \pi_{23} = \gamma'_{23} \cdot (1 - \nu_{23}) / \nu_{23} \). The numbers \( \pi_{12}, \pi_{23} \) represent the profit of the customers in the retail chain.

\[ \gamma'_{wj} \] is replaced by \( \gamma_{wj} = \gamma'_{wj} / \nu_{wj} \). The numbers \( \gamma \) and \( \gamma' \) differ in the units of measurement of the commodities delivered to the user \( j \). While \( \gamma' \) represents the sales at the cost, \( \gamma \) represents the same sales at actual selling prices.
In conclusion of this section, let us consider the numbers $\pi_{12}, \pi_{23}$ more closely. We see from the above discussion that the material costs are the only component of the costs of commodities sold for the customers in the retail chain; no other producing or transaction costs are considered. And yet in Section 4 the numbers $\pi_{12}, \pi_{23}$ are used as the admissible bounds on transaction costs, which are assumed to be unknown. It is in this sense we construct a model of a monotonic game of customers, Mullat [6, 1979].

3. Description of a retail chain: the general form

Consider now a retail chain consisting of $n$ participants indexed $w, j = 1,2,\ldots,n$. The state of a supplier $w$ is characterized by a $(m+1)$-component vector $\langle d_w, y_w \rangle = \langle d_w, \eta_{nk+1}, \ldots, \eta_{n1} \rangle$, $(n-k=m)$; the state of a customer $j$ by a $(v+1)$-component vector $\langle s_j, x_j \rangle = \langle s_j, \gamma_{1j}, \ldots, \gamma_{vj} \rangle$. The components of the $\langle d_w, y_w \rangle$ and $\langle s_j, x_j \rangle$ vectors are interpreted as follows: $d_w$ is the total orders amount of the supplier $w$ acting as a customer; $s_j$ is the capable sales total amount of the customer $j$ acting as a supplier; $\eta_{wj}$ is the cost of orders placed by the customer $j$ with the supplier $w$; $\gamma_{wj}$ are actual sales (deliveries) to customer $j$ from the supplier $w$. As indicated in the footnote, $\gamma_{wj}$ represents the deliveries valued at the selling prices of the customer $j$ acting as a supplier. The vectors $\langle d_w, y_w \rangle, \langle s_j, x_j \rangle$ are the order and the delivery vectors, respectively.

With each participant in the retail chain we associate certain domains in the nonnegative orthants $\mathbb{R}^{m+1}$ of the $(m+1)$- and $\mathbb{R}^{v+1}$ of the $(v+1)$- dimensional space. These domains $\mathbb{R}^{m+1}$ and $\mathbb{R}^{v+1}$ are the regions of feasible values of vectors $\langle d_w, y_w \rangle, \langle s_j, x_j \rangle$ in the $(m + v + 2)$-dimensional space.

For some of the participants vectors with $\gamma_{wj} > 0$ are inadmissible, and for some participants vectors with $\eta_{wj} > 0$ are inadmissible. Participants having the former property will be called produces and those having the latter property will be called distributors; all other participants in the retail chain will be called agents. In what follows the numbers $s_w$ ($w = 1,2,\ldots,k$) characterize the $k$ produces; the number $s_w$ represents the capable sales controlled by the participant $w$. The numbers $d_j$ ($j = v + 1, v + 2,\ldots,n$) correspondingly characterize the $r$ distributors: the number $d_j$ represents the demand to the external customers $(n - v = r)$.

Let us now impose certain constrains on the admissible vectors in this retail chain. The following constrains are strictly “local,” i.e., they apply to the individual participants in the retail chain.

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$k$ is the number of produces, see below.
The admissible retail chain states are constrained by balance conditions equating the actual sales from all the suppliers to a particular customer to capable sales of that customer acting as a supplier:

\[ s_j = \sum_{w=1}^{v} \gamma_{wj} \quad (j = k + 1, k + 2, \ldots, n). \]  

(1)

We also require balance conditions between the cost of orders placed by all the customers with a particular supplier and the demand figure of that supplier acting as a customer:

\[ d_w = \sum_{j=1}^{n} \eta_{wj} \quad (w = 1, 2, \ldots, v). \]  

(2)

As we have noted above, the retail chain considered in this article does not allow “closed-circuit motion” of orders or deliveries until a particular order reaches a producer or the delivery reaches a distributor. The indexes labeling the participants in such chains are ordered in a way 4 that if \( w \) is a supplier and \( j \) is a customer, then \( w < j \) (\( w = 1, 2, \ldots, v; j = v + 1, v + 2, \ldots, n \)). We call such chains as of a retail-type, and their description requires certain additional assumptions.

Consider the constants \( \alpha_{wj} \geq 0 \) and \( \beta_{wj} \geq 0 \) satisfying the following constraints:

\[ \sum_{j} \alpha_{wj} \leq 1 \quad (j > w; w = 1, 2, \ldots, v), \sum_{w} \beta_{wj} \leq 1 \quad (w < j; j = k + 1, \ldots, n) \]  

(3)

For the supplier \( w \), the number \( \alpha_{wj} \) is the fractional cost of orders made to the customer \( j \). For customer \( j \), the number \( \beta_{wj} \) is the fractional cost of the deliveries from supplier \( w \), which are necessary for meeting the sales target.

Suppose that purchase of orders in the retail chain move from distributors through agents to suppliers. This chain is conducted at the wholesale prices. The deliveries, also conducted at the wholesale prices of the chain in the opposite direction. We express the effective wholesale prices by a set of constants \( \nu_{wj} \) (\( w = 1, 2, \ldots, v; j = k + 1, k + 2, \ldots, n \)), which represent the participant’s cost per one bank note of sales for a customer acting as a supplier.

The set of constants \( \alpha_{wj}, \beta_{wj} \) and \( \nu_{wj} \) make it possible to uniquely determine the amount of orders and deliveries in a given transaction. Indeed, the amount of orders to the supplier \( w \) from the customer \( j \) is given by \( \eta_{wj} = \beta_{wj} \cdot d_j \cdot \nu_{wj} \). The relation (see Section 2) determines the amount of deliveries \( \gamma_{wj} = \min \{ \xi_{wj}, \eta_{wj} \} \), where \( \xi_{wj} = s_w \cdot \alpha_{wj} \) are the capable sales values at cost prices. Considering the difference in revenue from sales of customer \( j \) acting as a supplier, we conclude that the deliveries from the supplier \( w \) to the customer \( j \) are given by \( \gamma_{wj} = \gamma_{wj} / \nu_{wj} \).

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4 The term topological sorting originates from Knuth [3, 1972] to describe the ordering of indexes having this property.
In conclusion, let us consider one computational aspect of order and delivery vectors in a retail-type distribution chain. It is easily seen that the components $d_j$, $s_w$, $\eta_{wj}$ and $\gamma_{wj}$ ($w = 1,2,...,v; j = k + 1,k + 2,...,n$) as obtained from (1) and (2) are given by

$$d_w = \sum_j \beta_{wj} \cdot d_j \cdot v_{wj} \quad (j > w; w = 1,2,...,v)$$ (4)

$$s_j = \sum_w \min\{s_w \cdot \alpha_{wj}; \beta_{wj} \cdot d_j \cdot v_{wj}\} / v_{wj} \quad (w < j; j = k + 1,...,n)$$ (5)

The starting data in (4) is the demand of the distributors to external customers, i.e., the numbers $d_{v+1},d_{v+2},...,d_n$. The starting data in (5) are the capable sales amounts $s_1,s_2,...,s_k$ of the produces, which together with the numbers $d_1,d_2,...,d_v$ from (4) are used in (5) to compute the actual sales of the customers.

4. A monotonic game of customers in the retail chain

In the previous section we considered a retail-type distribution in the chain with participants indexed by $w = 1,2,...,v; j = k + 1,k + 2,...,n$: the index $j$ identifiers a customer, the index $w$ identifiers a supplier.

Let us interpret the activity of the retail chain as a monotonic game, Mullat [6, 1979], in which the customers need to decide from what supplier to order a particular commodity.

Suppose that in addition to the cost of materials, the customers bear uncertain transaction costs in their bilateral trade with suppliers. Because of the uncertainty of transaction costs, it is quite possible that in some transactions the costs will exceed the gross profit from sales. In this case, the potentially feasible transactions will not take place.

Let the set $R_j$ represents all the potential transactions corresponding to the set of suppliers from which the customer $j$ is to make his choice. The choice of the customer $j$ ($j = k + 1,k + 2,...,n$) is a subset $A^j$ of the set $R_j : A^j \subseteq R_j$; the case $A^q = \emptyset$ is not excluded: it requires the customer’s refusal to make a choice. The collection $\{A^{k+1},A^{k+2},...,A^n\}$ represents the customer’s joint choice. It is readily seen that the sets $R_j$ are finite and nonintersecting; their union corresponds to set $W = R_{k+1} \cup R_{k+1} \cup ... \cup R_n$.

In what follows, we focus on the criterion by which the customer $j$ chooses his suppliers $A^j$ while the lowest transaction costs, as a threshold $u^p$, increases. In contrast to the standard monotonic game, Mullat [6, 1979], which is based on a coalition formation, we will consider the strategy of individual customers whose objective is to maximize the profit from the actual sales revenues. We will thus essentially deal with $m$ players’ game, $m = n - k$.

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5 Here we need only consider the principles of the computational procedure.
Let us first introduce a measure of the utility of a transaction between customer \( j \) and supplier \( w \in A^j \) \((j = k + 1, k + 2, \ldots, n)\). The utility of a transaction between customer \( j \) and supplier \( w \) is expressed by the corresponding profit \( \pi_{wj} = \gamma_{wj} \cdot (1 - \nu_{wj}) \).

The utility of a transaction with a supplier \( w \in A^j \) is a function \( \pi_{wj}(X_{k+1}, X_{k+2}, \ldots, X_n) \) of many variables: the value of the variable \( X_j \) is the choice \( A^j \) of the customer \( j \), the number of variables is \( m = n - k \). To establish this fact, it is sufficient to show how to compute the components of the order and delivery vectors from the joint choice \( \langle X_{k+1}, X_{k+2}, \ldots, X_n \rangle \). Indeed, according to our description, a retail-type distribution in the chain requires defining the constants \( \alpha_{wj} \geq 0 \) and \( \beta_{wj} \geq 0 \) \((w = 1,2,\ldots,v; j = k + 1,\ldots,n)\) that satisfy the constraints \( 3 \). A pair of constants \( \alpha_{wj} \) and \( \beta_{wj} \) can be assigned in a one-to-one correspondence to a supplier \( w \in R_j \), rewriting \( 3 \) in the form

\[
\sum_{w \in R_j} \alpha_{wj} \leq 1 \ (w = 1,2,\ldots,v), \quad \sum_{w \in R_j} \beta_{wj} \leq 1 \ (j = k + 1,\ldots,n) \tag{6}
\]

If the constrains \( 6 \) are satisfied, then the same constrains are of necessity satisfied on the subsets \( A^j \) of the set \( R_j \). Thus, restricting \( 4 \) and \( 5 \) to the sets \( X_j \subseteq R_j \), the numbers \( \gamma_{wj} \) can be uniquely calculated for every joint choice \( \langle X_{k+1}, X_{k+2}, \ldots, X_n \rangle \). Finally, let us define the individual utility criterion of the customer \( j \) in the form

\[
\Pi_j = \sum_{w \in A^j} (\pi_{wj} - u_{wj}), \tag{7}
\]

where \( u_{wj} \) are the customer \( j \) transaction costs allocable to the supplier \( w \in A^j \); we define \( \Pi_j = 0 \) if the customer \( j \) refused to make a choice — \( A^j = \emptyset \).

The function \( \pi_{wj}(X_{k+1}, X_{k+2}, \ldots, X_n) \) has the obvious property of monotone utility, so that for every pair of joint choices of customers \( \langle L^{k+1}, L^{k+2}, \ldots, L^n \rangle \) and \( \langle G^{k+1}, G^{k+2}, \ldots, G^n \rangle \) such that \( L^j \subseteq G^j \) \((j = k + 1,\ldots,n)\) we have

\[
\pi_{wj}(L^{k+1}, L^{k+2}, \ldots, L^n) \leq \pi_{wj}(G^{k+1}, G^{k+2}, \ldots, G^n). \tag{8}
\]

The property of monotone utility leads to certain conclusions concerning the behavior of customers depending on the individual utility criterion. Under certain conditions, rational behavior of customer \( j \) (i.e., maximization of the profit \( \Pi_j \)) is equivalent to avoid profit-loosing transaction with all the suppliers \( w \in A^j \). This aspect is not made explicit in Mullat [7, 1979], although it is quite obvious. Thus, using the lemma, see the English version at p.1473, we can easily show that if the utilities \( \pi_{wj}(\ldots, X_j, \ldots) \) are independent of the choice \( X_j \), the customer \( j \) maximizes his profit \( \Pi_j \) by extending his choice to the set-theoretically largest choice. In what follows we will show that this result also applies under a weaker assumptions.
First, a few reservations about the proposed condition – see (9) below. This condition has a simple economic meaning: the customer \( j \) entering into loosing transactions cannot achieve a net increase in his utility of the losses. For example, if for fixed choices of all other customers in the retail chain, the utilities \( \pi_{wj}(...,X_j,...) \) for \( w \in X_j \) are independent of the choice \( X_j \), the condition (9) hold as strict inequalities. These conditions are also reduces to strict inequalities when, for instance, the capable sales \( \xi_{wj} \) in each transaction between customer \( j \) and supplier \( w \in A^j \) is not less than the demand \( \eta_{wj} \) so that every customer can receive the entire quantity ordered from his suppliers. In particular, by increasing the producers’ supply \( s_1, s_2, ..., s_k \) with unlimited manufacturing capacity, we can always increase the capable sales to such an extent that it exceeds the demand, so that the conditions (9) are satisfied.

We can now formulate the final conclusion: the following lemma suggests that each customer will make his choice so as to maximize the profit \( \Pi_j \), providing all the other customers keep their choices fixed.\(^6\)

Let the suppliers not entering the set \( A_j \) be assigned indexes \( q = 1, 2, ... \). Then the profit \( \Pi_j \) of customer \( j \) is represented by a many-variable function \( \Pi_j(t_{1j}, t_{2j}, ...) \) with variables \( t_{qj} \) varying on \([0, \beta_{qj}]\).\(^7\) The value of the function \( \Pi_j(t_{1j}, t_{2j}, ...) \) is the customer’s profit for the case when the customer \( j \) has extended the choice by placing orders in the amounts of \( t_{qj}, d_j \cdot v_{qj} \) with the suppliers \( q = 1, 2, ... \) outside the choice \( A_j \). Thus the set of variables \( t_{qj} \) identifiers the suppliers \( q = 1, 2, ..., \), and customers \( j \) who expand their choice \( A_j \). If all \( t_{qj} = 0 \), the choice \( A_j \) is not expanded and the profit \( \Pi_j(0, 0, ...) \) coincides with (7).

The profit function \( \Pi_j(t_{1j}, t_{2j}, ...) \) thus has to satisfy the following constraint: for every \( t_{qj} \) in \([0, \beta_{qj}]\) \( q = 1, 2, ... \)

\[
\Pi_j(t_{1j}, t_{2j}, ...) \leq \Pi_j(0, 0, ...) \tag{9}
\]

**Definition.** A joint choice \( \langle A_{o}^{k+1}, ..., A_{o}^{n} \rangle \) of the retail chain customers is said to be rational with the threshold \( u^o \) if, given a amount of transaction costs not less than \( u^o > 0 \), the utility measure \( \pi_{wj} \geq u^o \) in every transaction of customer \( j \) with the supplier \( w \in A_j \) ( \( j = k + 1, ..., n \)).

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\(^6\) The joint choice of users having this property is generally interpreted in the sense of Nash equilibrium, [8, 1953], see also Owen, [9, 1968].

\(^7\) We recall that \( \beta_{qj} \) is the fractional cost of all the orders placed with supplier \( q \).
Lemma. The set-theoretically largest choice \( S^o = \left\{ A_{k+1}^o, ..., A_n^o \right\} \) among all the joint choices rational with threshold \( u^o > 0 \) ensures that the retail-type distribution chain is in equilibrium relative to the individual profit criterion \( \Pi_j \) under the following conditions: a) the transaction costs \( u_{wj} \) for \( w \in S^o \) do not exceed \( \min \pi_{wj} \) over \( w \in S^o \cap R_j \); b) inequality (9) holds.

Proof. Let \( S^o \) be a set-theoretically largest choice among all the joint choices rational with the threshold \( u^o \), i.e., \( S^o \) is the largest choice \( H \) among all the choices such that \( \pi_{wj} (H \cap R_{k+1}, ..., H \cap R_n) \geq u^o \). Suppose that some customer \( p \) achieves a profit higher than \( \Pi_p \) by making the choice \( A^p \subseteq R_p \), which is different from \( S^o \cap R_p \): \( \Pi_p' = \sum_{w \in A^p} (\pi_{wp}(..., A^p, ...,) - u_{wp}) > \Pi_p \), subject to \( u^o \leq u_{wp} \leq \min \pi_{wp} \). Clearly, the choice \( A^p \) is not a subset of \( S^o \), since this would contradict the monotone property (8), so that \( A^p \setminus S^o \neq \emptyset \). By the same monotone property, the customer making the choice \( A^p \cup (S^o \cap R_p) \) will achieve a profit not less than \( \Pi_p' \). On the other hand, all transactions in \( A^p \setminus S^o \) are losing transactions for this customer, since \( S^o \) is the set-theoretically largest set of non-losing bilateral trade agreements tolerant towards the transactions costs’ threshold \( u^o > 0 \). For the customer \( p \) making the choice \( A^p \cup (S^o \cap R_p) \) the profit \( \Pi_p' \) does not decrease only if the total increase in utility due to the contribution \( \pi_{wp} \) of the transactions \( w \in S^o \cap R_p \) exceeds the total negative utility due to the transactions in \( A^p \setminus S^o \). Clearly, because of the constraint (9), the customer \( p \) has no such an opportunity. This contradiction establishes the truth of the lemma.

In conclusion, we would like to consider yet another point. With uncertain transaction costs, the refusal to enter into any transaction may lead to an undesirable “snowballing” of refusals by customers to choose their suppliers. It therefore seems that customers will attempt at least to conclude transactions with \( \pi_{wj} \geq u^o \), even when there is some risk that the transaction costs will exceed the utility \( \pi_{wj} \). Thus, without exaggeration, we may apparently state that the size of the interval \([u^o, \min \pi_{wj}]\) reflects the elasticity of the customer’s choice: the number \( \min \pi_{wj} - u^o \) is thus a measure of a “risk” that the customer will get into non-equilibrium situation. Clearly, a customer with a small interval will have greater difficulties to maintain the equilibrium than a customer with a wide interval.

5. Final remarks

It ends where we started. The paper investigated a situation of distributing commodities in the retail chain with participants making “to buy and sales” decisions in a retail chain. One type of participants’ produce and sale, others buy and sale, the third only buy for consumption. The price system was set up via some constants, which are nothing but percentages to
perform calculus of how the sales price must depend and exceed the purchasing prices to achieve a satisfactory results for participants maximizing their profits. The situation becomes complex as soon as to buy and sale decisions incorporated transaction costs. Transaction costs interact into the behavior of participants by transforming potentially profitable into loosing transactions. The paper investigated the situation, as global, depending on the transaction costs' threshold varying the threshold from low to high values until all transactions, allegedly profitable in bilateral trade agreements, became loosing and do not any more form a basis for an agreement between rational participants. The retail chain structure, while the transaction costs' threshold is increasing, changes like nested set of retail chains each of them on the higher threshold is capable to counteract higher transaction costs and still functioning in equilibrium. Condition for such a rational behavior was that all participants in the retail chain must avoid any loosing transaction. Beyond the goal of the retail chain formation to hold the retail chain in equilibrium, some elasticity intervals for transaction costs, where it still was realistic to buy and sale rationally, have been internally encoded into the scheme and calculated individually for all participants in the chain.

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