Equilibrium in a Distribution Network with Transaction Costs

JOSEPH E. MULLAT [*]

Department of Economics, Tallinn Technical University (1973 – 1980)

Abstract

The paper addresses a situation of how the network consisting of suppliers, agents, and distributors transforms while the costs of transactions increase. The stream of orders and deliveries between the relevant interest groups results in a nested chain of coalitions if the participants follow the rules of monotonic game with transaction cost as a parameter, Mullat (1979). The smallest coalition in the chain is the most tolerant coalition towards the costs. The participants making to buy and sale decisions remain in equilibrium under condition that the gains of trade exceed transaction costs.

Keywords: suppliers, distributors, monotonic game, network

> *Businessmen in deciding on their ways of doing business and on what to produce have to take into account transaction costs. If the cost of making an exchange are greater than the gains which that exchange would bring, that exchange would not take place and the greater production that would flow from specialization would not be realized. In this way transaction costs affect not only contractual arrangements, but also what goods and services are produced.* Ronald H. Coase, "The Institutional Structure of Production," Ménard, C., and M. M. Shirley (eds.) (2005), *Handbook of New Institutional Economics*, Spriner: Dordrecht, Berlin, Heidelberg, New York. XIII. 884pp., p.35, ISBN 1-4020-2687-0.

1. **Introduction**

All, perhaps, know that prices on commodity markets sometimes continue to rise unabated on the back of an anticipated shortage in the global raw materials availability and sharp volatility in the commodity future markets and terminal prices on fears of an immediate shortage of materials in the short term. Along with the significant increase in commodity prices, on the one hand, the costs increase on inputs like petroleum, electricity, etc., while currency exchange rates also moving adversely on the other, the situation becomes uncertain. As an example, one may point at recent market price increase of coffee raw materials, which did have immediate consequences for some known positions in distribution network while the distributors, nonetheless, demonstrate readiness to make loosing transactions. It is also understandable that it would be impossible for the distributor to take frequent price changes in the market place again and again. With this in mind, distributors are trying to hold prices constant. However, given the current context, they will have no other option but to seek a price change for distributed commodities with immediate effect.

Uncertainties in market prices of commodities always lead to an increase of overhead expenses or transaction costs. Transaction costs increase once again leads to additional uncertainties, and the distributors in the network end up in a dead circle of price increase, which may result that the exchange does not take place, and the market old supply and demand structure to be replaced with a new. In the environment of constant price increase, the orders and deliveries will do not match any more for a given supply and demand structure. In such situations, individual participants in the network are still assumed to act rationally finding a new ways of making business with the object of maximizing the profit by

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[*] Independent researcher, docent in the period 1979 – 1980; Byvej 269, 2650 Hvidovre, Denmark, mailto: jm@mail-telia.dk

trying to restructure the network. Worth to note that New Institutional Economics gives an explanation for transactions as mediated through the market in two directions: the vertical integration, Joskow (2005), where the market structure is mostly an internal production stream of services and product components, and the horizontal stream of services and products outsourced by companies if needed to produce the end product.

This paper addresses the above situation in question by setting up a coalition game of the participants in the network grounding on supposition that orders and deliveries be met with uncertainty of transaction costs. In so doing, the paper attempts to develop a numerical description of the supply and demand structure for the deliveries of commodities in the network. The allegedly rational behavior of a participant is not always such, because the participants on purpose may attempt to enter irrationally into certain losing transactions in hope to offset the negative effect of the former. Given this irrational situation the prices will increase additionally upon already profitable transactions. Numerical analysis of irrational situations reveals, however, that in case the participants will try to avoid all losing transactions, their behavior is once again becoming rational and the participants of the network in such situations will end up in the Nash equilibrium (1953).

To our knowledge (or lack of that), the coalition formation, or in mundane terms the restructuring process of the network, is rather complicated mathematical problem, which do not have satisfactory solutions. However, in recent years it has become clear that a mathematical structure known as antimatroid is well suited for such type a coalition formation process, Algaba, et al. (2004). Antimatroid is a collection of potential coalitions – subsets of participants, i.e. those who make decisions to buy and sale in bilateral market transactions. That is to say, one will always find a path of bilateral transactions connecting members of the coalition with each other by mutual business interests in the subset belonging to antimatroid making exchange in a characteristic coalition, if the latter forms of course.

We step up beyond convention of the theory of coalition games that the solution mandatory has to be a core, and take the coalition formation process in terms of so-called defining sequence of transactions, Mullat (1979). The sequence facilitates the coalition formation as a transformation process of nested transactions, which ends at its last and highest level with the coalition named kernel. The kernel operates as a coalition of members capable to cover the highest costs of transactions in case of uncertainty. Defining sequence of transactions actually produces an antimatroid of subsets proved by Levit and Zaks (2001) at greedoids; what is a greedoid see also Korte et al, (1991). The sequence on antimatroid extends, c.f. Rapoport (1985), Shapley's value heuristic procedure of greedy type but in inverse order.

Bearing all this in mind the suggested framework allows to perform a series of computer simulations. First, in order to determine the possible response of the network participants to different supply and demand structures. Second, in order to identify the participants where the executive efforts might be applied to prevent distinctive actions that may misbalance the equilibrium in the network. With this object, we used a model to construct an "elasticity" measure for the choice of customers; this measure is represented by the overhead expense interval – transaction costs for which the network remains in equilibrium.

The rest of this paper is structured as follows. The next section sets up the basic concepts intending to bring at the surface the calculus of utilities of participants in the network. It is a preliminary step necessary to move forward to the Section 3, where the general model of participants of the network is described. In Section 4, which is main part of the paper, a coalition game of customers addresses the process of coalition formation in details. Here the monotonic property of utilities plays its major role. A summary of the results ends the study.

2. Description of a distribution network: the stream model

To consider the simplest case of commodities distribution in a stream might be instructive. This elementary model is used at current stage solely as a convenient means of simplifying the presentation.

The distribution of commodities in the network is characterized by sales figures that may be expressed as one of the following three alternative numbers: a) a demand η which is disclosed to the particular participant either externally or by other participant in the network; b) a capable supply ξ calculated at the cost of all commodities produced by the participant for delivery outside the network or to the other participants; c) actual sales γ calculated at the prices actually paid by the customers for the delivered commodities.

An order is thus defined as a certain quantity of a particular commodity ordered by one of the participant's from another participant in the network; a delivery is similarly defined as a certain quantity of a commodity delivered by one of the participant's to another participant in the network. We assume that the network includes suppliers who are only capable of making deliveries – the produces; participants, who both issue orders and make deliveries – the agents; and the distributors, who only order commodities from other participants. [1]

In what follows we always consider the stream of orders and deliveries for the case of stream or "pipeline" distribution without "closed circuits." Therefor, we can always identify a unique direction of "stream" of orders from the distributors to the produces via agents and a "stream" of deliveries in the reverse direction.

Let us consider in more detail this particular stream of orders and deliveries of commodities in the network. The direction of the stream of orders (deliveries) is defined by assigning serial numbers – the indexes $1{,}2$ and 3 – to the producer, to the agent, and to the distributor, respectively. The producer and the agent act as suppliers, the agent and the distributor act as customers. The agent thus has the dual role of a supplier and a customer, whereas the producer only acts as a supplier and the distributor only acts as a customer.

The stream of orders to the produces from the customers is characterized by two numbers η_{23} and η_{12}. The number η_{wj} $\left(w = 1{,}2;\, j = 2{,}3\right)$ is the demand η_{wj} disclosed by the customer j to the supplier w. We assume that sales equal the distribution in this network. Two numbers ξ_{12} and ξ_{23}, which are interpreted as the corresponding capable sales similarly characterize the stream of deliveries to the distributor. We assume that sales equal the distribution in this network.

Suppose that the demand of the distributor to the external customers is fixed at the level of d bank notes. The capable sales of the producer are s bank notes. In other words, d is the estimated level of orders from the external customers and it plays the same role as the number η for the customers in the network. Similarly, s is the intrastate level of estimated deliveries by the producer, and it has the same role as ξ for the customers.

Let us now consider the exact situation in a stream. To make deliveries at a demand level of d bank notes, the distributor have to place orders with the agent in the amount of $\eta_{23} = \nu_{23} \cdot d$ bank notes, where ν_{23} are the distributor's cost of commodities sold (the cost per 1 bank note of sales). The agent, having received an order from the distributor, will in

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[1] Note that in subsequent sections distributors naturally also act as suppliers to external customers.

turn place an order with the supplier in the amount $\nu_{12} \cdot \eta_{23}$, where ν_{12} is the agent's cost per 1 bank note of sales. On the other hand, the estimated sales of the producer are ξ_{12} bank notes, $\xi_{12} = s$. Assuming that all the transactions between the suppliers and the customers in the network are materialized in amounts not less than those indicated in the purchase orders, the actual sales of the producer to the agent are given by $\gamma'_{12} = min\{\xi_{12}, \eta_{12}\}$.

Now, since the agent paid the producer γ'_{12} for the commodities ordered, the agent's revenue is $\xi_{23} = \gamma'_{12}/\nu_{12}$, where clearly $\xi_{23} \geq \gamma'_{12}$. The difference between the revenue ξ_{23} and the costs γ'_{12} is defined as $\pi_{12} = \gamma'_{12} \cdot (1 - \nu_{12})/\nu_{12}$.

From the same considerations, $\gamma'_{23} = min\{\xi_{23}, \eta_{23}\}$ [2] give the actual sales of the agent to the distributor. We similarly define the difference $\pi_{23} = \gamma'_{23} \cdot (1 - \nu_{23})/\nu_{23}$. The numbers π_{12}, π_{23} represent the profit of the customers in the network.

In conclusion of this section, let us consider the numbers π_{12}, π_{23} more closely. We see from the above discussion that the material costs are the only component of the costs of commodities sold for the customers in the network; no other producing costs and no overhead expenses are considered. And yet in Section 4 the numbers π_{12}, π_{23} are used as the admissible bounds on overhead expenses, which are assumed to be unknown. It is in this sense we construct a model of a monotone game of customers.

3. Description of a distribution network: the general form

Consider now a distribution network consisting of n participants indexed $w,\ j = 1,2,...,n$. The state of a supplier w is characterized by a $(m+1)$-component vector [3] $\langle d_w, y_w \rangle = \langle d_w, \eta_{wk+1,...,}\eta_{wn} \rangle$, $(n - k = m)$; the state of a customer j by a $(\nu+1)$-component vector $\langle s_j, x_j \rangle = \langle s_j, \gamma_{1j}, ..., \gamma_{\nu j} \rangle$. The components of the $\langle d_w, y_w \rangle$ and $\langle s_j, x_j \rangle$ vectors are interpreted as follows: d_w is the total orders amount of the supplier w acting as a customer; s_j is the capable sales total amount of the customer j acting as a supplier; η_{wj} is the cost of orders placed by the customer j with the supplier w; γ_{wj} are actual sales (deliveries) to customer j from the supplier w. As indicated in the footnote, γ_{wj} represents the deliveries valued at the selling prices of the customer j acting as a supplier. The vectors $\langle d_w, y_w \rangle$, $\langle s_j, x_j \rangle$ are the order and the delivery vectors, respectively.

With each participant in the network we associate certain domains in the nonnegative orthants $\Re^{m+1}$ of the $(m+1)$ – and $\Re^{\nu+1}$ of the $(\nu+1)$ – dimensional space. These domains $\Re^{m+1}$ and $\Re^{\nu+1}$ are the regions of feasible values of vectors $\langle d_w, y_w \rangle$, $\langle s_j, x_j \rangle$ in the $(m+\nu+2)$ – dimensional space.

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[2] In subsequent sections, γ'_{wj} is replaced by $\gamma_{wj} = \gamma'_{wj}/\nu_{wj}$. The numbers γ and γ' differ in the units of measurement of the commodities delivered to the user j . While γ' represents the sales at the cost, γ represents the same sales at actual selling prices.

[3] k is the number of produces, see below.

For some of the participants vectors with $\gamma_{wj} > 0$ are inadmissible, and for some participants vectors with $\eta_{wj} > 0$ are inadmissible. Participants having the former property will be called produces and those having the latter property will be called distributors; all other participants in the network will be called agents. In what follows the numbers s_w ($w = 1,2,...,k$) characterize the k produces; the number s_w represents the capable sales controlled by the participant w. The numbers d_j ($j = v+1, v+2,...,n$) correspondingly characterize the r distributors: the number d_j represents the demand to the external customers ($n - v = r$).

Let us now impose certain constrains on the admissible vectors in this network. The following constrains are strictly "local," i.e., they apply to the individual participants in the network.

The admissible network states are constrained by balance conditions equating the actual sales from all the suppliers to a particular customer to capable sales of that customer acting as a supplier:

$$s_j = \sum\nolimits_{w=1}^{v} \gamma_{wj} \quad (j = k+1, k+2,...,n). \tag{1}$$

We also require balance conditions between the cost of orders placed by all the customers with a particular supplier and the demand figure of that supplier acing as a customer:

$$d_w = \sum\nolimits_{j=i+1}^{n} \eta_{wj} \quad (w = 1,2,...,v). \tag{2}$$

As we have noted above, the distribution network considered in this article does not allow "closed-circuit motion" of orders or deliveries until a particular order reaches a producer or the delivery reaches a distributor. The indexes labeling the participants in such networks are ordered so [4] that if w is a supplier and j is a customer, then $w < j$ ($w = 1,2,...,v;\ j = v+1, v+2,...,n$). Such networks are called stream-type, and their description requires certain additional assumptions.

Consider the constants $\alpha_{wj} \geq 0$ and $\beta_{wj} \geq 0$ satisfying the following constraints:

$$\sum_j \alpha_{wj} \leq 1 \ (j > w; w = 1,2,...,v), \ \sum_w \beta_{wj} \leq 1 \ (w < j;\ j = k+1,...,n) \tag{3}$$

For the supplier w, the number α_{wj} is the fractional cost of orders made to the customer j. For customer j, the number β_{wj} is the fractional cost of the deliveries from supplier w which are necessary for meeting the sales target.

Suppose that purchase of orders in the distribution network move from distributors through agents to suppliers. This stream is conducted at the wholesale prices. The deliveries (also conducted at the wholesale prices) stream in the opposite direction. We express the effective wholesale prices by a set of constants v_{wj} ($w = 1,2,...,v;\ j = k+1, k+2,...,n$), which represent the participant's cost per one bank note of sales for a customer acting as a supplier.

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[4] The term topological sorting originates from Knuth (1972) to describe the ordering of indexes having this property.

The set of constants α_{wj}, β_{wj} and ν_{wj} make it possible to uniquely determine the level of orders and deliveries in a given transaction. Indeed, the level of orders to the supplier w from the customer j is given by $\eta_{wj} = \beta_{wj} \cdot d_j \cdot \nu_{wj}$. The relation (see Section 2) determines the level of deliveries $\gamma'_{wj} = min\{\xi_{wj}, \eta_{wj}\}$, where $\xi_{wj} = s_w \cdot \alpha_{wj}$ are the capable sales values at cost prices. Considering the difference in revenue from sales of customer j acting as a supplier, we conclude that the deliveries from the supplier w to the customer j are given by $\gamma_{wj} = \gamma'_{wj} / \nu_{wj}$.

In conclusion of this section, let us consider one computational aspect of order and delivery vectors in a stream-type distribution network.[5] It is easily seen that the components d_j, s_w, η_{wj} and γ_{wj} ($w = 1,2,...,v; j = k+1, k+2,...,n$) as obtained from (1) and (2) are given by

$$d_w = \sum_j \beta_{wj} \cdot d_j \cdot \nu_{wj} \quad (j > w; w = 1,2,...,v) \tag{4}$$

$$s_j = \sum_\omega min\{s_w \cdot \alpha_{wj}; \beta_{wj} \cdot d_j \cdot \nu_{wj}\} / \nu_{wj} \quad (w < j; j = k+1,...,n) \tag{5}$$

The starting data in (4) is the demand of the distributors to external customers, i.e., the numbers $d_{v+1}, d_{v+2},...,d_n$. The starting data in (5) are the capable sales levels $s_1, s_2,...,s_k$ of the produces, which together with the numbers $d_1, d_2,...,d_v$ from (4) are used in (5) to compute the actual sales of the customers.

4. A monotonic game of customers in the distribution network

In the previous section we considered a stream-type distribution network with participants indexed by $w = 1,2,...,v; j = k+1, k+2,...,n$. The index j identifiers a customer, the index w identifiers a supplier.

Let us interpret the activity of the network as a monotone game, Mullat (1979), in which the customers need to decide from what supplier to order a particular commodity.

Suppose that in addition to the cost of materials, the customers bear uncertain overhead costs (transaction costs) in their transactions with the suppliers. Because of the uncertainty of overheads, it is quite possible that in some transactions the overheads will exceed the gross profit from sales. In this case, the potentially feasible transactions will not take place.

Let the set R_j represents all the potential transactions corresponding to the set of suppliers from which the customer j is to make his choice. The choice of the customer j ($j = k+1, k+2,...,n$) is a subset A^j of the set R_j : $A^j \subseteq R_j$; the case $A^q = \varnothing$ is not excluded: it requires the customer's refusal to choose. The collection $\langle A^{k+1}, A^{k+2},...,A^n \rangle$ represents the customer's joint choice. It is readily seen that the sets R_j are finite and nonintersecting; their union corresponds to set W : $W = R_{k+1} \cup R_{k+1} \cup ... \cup R_n$.

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[5] Here we need only consider the principles of the computational procedure.

In what follows, we focus on the criterion by which the customer j chooses his suppliers A^j. In distinction from the standard monotone game, Mullat (1979), which is based on a coalition of players, we will consider the strategy of individual customers whose objective is to maximize the profit from the actual sales revenues. We will thus essentially deal with a coalition-less m players game, $m = n - k$.

Let us first introduce a measure of the utility of a transaction between customer j and supplier $w \in A^j$ ($j = k+1, k+2, ..., n$). The utility of a transaction between customer j and supplier w is expressed by the corresponding profit $\pi_{wj} = \gamma_{wj} \cdot (1 - \nu_{wj})$.

The utility of a transaction with a supplier $w \in A^j$ is a function $\pi_{wj}(X_{k+1}, X_{k+2}, ..., X_n)$ of many variables: the value of the variable X_j is the choice A^j of the customer j, the number of variables is $m = n - k$. To establish this fact, it is sufficient to show how to compute the components of the order and delivery vectors from the joint choice $\langle X_{k+1}, X_{k+2}, ..., X_n \rangle$. Indeed, according to our description, a stream-type distribution network requires defining the constants $\alpha_{wj} \geq 0$ and $\beta_{wj} \geq 0$ ($w = 1,2,...,v; j = k+1,...,n$) that satisfy the constraints (3). A pair of constants α_{wj} and β_{wj} can be assigned in a one-to-one correspondence to a supplier $w \in R_j$, rewriting (3) in the form

$$\sum_{w \in R_j} \alpha_{wj} \leq 1 \ (w = 1,2,...,v), \ \sum_{w \in R_j} \beta_{wj} \leq 1 \ (j = k+1,...,n) \tag{6}$$

If the constrains (6) are satisfied, then the same constrains are of necessity satisfied on the subsets A^j of the set R_j. Thus, restricting (4) and (5) to the sets $X_j \subseteq R_j$, the numbers γ_{wj} can be uniquely calculated for every joint choice $\langle X_{k+1}, X_{k+2}, ..., X_n \rangle$. Finally, let us define the individual utility criterion of the customer j in the form

$$\Pi_j = \sum_{w \in A^j} (\pi_{wj} - u_{wj}), \tag{7}$$

where u_{wj} are the customer's overhead expenses allocable to his transaction with the supplier $w \in A^j$; we define $\Pi_j = 0$ if the customer refused to choose - $A^j = \varnothing$.

The function $\pi_{wj}(X_{k+1}, X_{k+2}, ..., X_n)$ has the obvious property of monotone utility, so that for every pair of joint choices of customers $\langle L^{k+1}, L^{k+2}, ..., L^n \rangle$ and $\langle G^{k+1}, G^{k+2}, ..., G^n \rangle$ such that $L^j \subseteq G^j$ ($j = k+1,...,n$) we have

$$\pi_{wj}(L^{k+1}, L^{k+2}, ..., L^n) \leq \pi_{wj}(G^{k+1}, G^{k+2}, ..., G^n). \tag{8}$$

The property of monotone utility leads to certain conclusions concerning the behavior of customers depending on the individual utility criterion. Under certain conditions, rational behavior of customer j (i.e., maximization of the profit Π_j) is equivalent to loss avoidance in every individual transaction with the supplier $w \in A^j$. This aspect is not made explicit in

Mullat (1979), although it is quite obvious. Thus, using the lemma in the same paper (see the English version at p.1473) we can easily show that if the utilities $\pi_{wj}(...,X_j,...)$ are independent of the choice X_j, the customer j maximizes his profit Π_j by extending his choice to the set-theoretically largest choice. In what follows we will show that this result also applies under a weaker assumption.

First, a few reservations about the proposed condition – see (9) below. This condition has a simple economic meaning: the customer j entering into loosing transactions cannot achieve a net increase in his utility of the losses. For example, if for fixed choices of all other customers in the network, the utilities $\pi_{wj}(...,X_j,...)$ for $w \in X_j$ are independent of the choice X_j, the condition (9) hold as strict inequalities. These conditions are also reduces to strict inequalities when, for instance, the capable sales ξ_{wj} in each transaction between customer j and supplier $w \in A^j$ is not less than the demand η_{wj} so that every customer can receive the entire quantity ordered from his suppliers. In particular, by increasing the producers' supply $s_1, s_2, ..., s_k$ with unlimited manufacturing capacity, we can always increase the capable sales to such an extent that it exceeds the demand, so that the conditions (9) are satisfied.

We can now formulate the final conclusion: the following lemma suggests that each customer will make his choice so as to maximize the profit Π_j, providing all the other customers keep their choices fixed.[6]

Let the suppliers not entering the set A_j be assigned indexes $q = 1,2,...$. Then the profit Π_j of customer j is represented by a many-variable function $\Pi_j(t_{1j}, t_{2j}, ...)$ with variables t_{qj} varying on $[0, \beta_{qj}]$.[7] The value of the function $\Pi_j(t_{1j}, t_{2j}, ...)$ is the customer's profit for the case when the customer j has extended the choice by placing orders in the amounts of $t_{qj} \cdot d_j \cdot v_{qj}$ with the suppliers $q = 1,2,...$ outside the choice A_j. Thus the set of variables t_{qj} identifiers the suppliers $q = 1,2,...$, and customers j who expand their choice A_j. If all $t_{qj} = 0$, the choice A_j is not expanded and the profit $\Pi_j(0,0,...)$ coincides with (7).

The profit function $\Pi_j(t_{1j}, t_{2j}, ...)$ thus has to satisfy the following constraint: for every t_{qj} in $[0, \beta_{qj}]$ $q = 1,2,...$

$$\Pi_j(t_{1j}, t_{2j}, ...) \le \Pi_j(0,0,...) \tag{9}$$

Definition. *A joint choice* $\left\langle A_o^{k+1}, ..., A_o^n \right\rangle$ *of the network customers is said to be rational with the threshold* u^o *if, given a level of overhead expenses not less than* $u^o > 0$, *the utility measure* $\pi_{wj} \ge u^o$ *in every transaction of customer* j *with the supplier* $w \in A_o^j$ ($j = k+1, ..., n$).

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[6] The joint choice of users having this property is generally interpreted in the sense of Nash equilibrium, (1953), see also Owen, (1968).

[7] We recall that β_{qj} is the fractional cost of all the orders placed with supplier q.

Lemma. *The set-theoretically largest choice* $S^o = \left\langle A_o^{k+1},\dots,A_o^n \right\rangle$ *among all the joint choices rational with threshold* $u^o > 0$ *ensures that the stream-type distribution network is in equilibrium relative to the individual profit criterion* Π_j *under the following conditions: a) the overhead expenses* u_{wj} *for* $w \in S^o$ *do not exceed* $min\,\pi_{wj}$ *over* $w \in S^o \bigcap R_j$*; b) inequality (9) holds.*

The proof is given in the appendix.

In conclusion, we would like to consider yet another point. With uncertain overhead expenses, the refusal to enter into any transaction may lead to an undesirable "snowballing" of refusals by customers to choose their suppliers. It therefore seems that customers will attempt at least to conclude transactions with $\pi_{wj} \geq u^o$, even when there is some risk that the overhead expenses will exceed the utility π_{wj}. Thus, without exaggeration, we may apparently state that the size of the interval $[u^o, min\,\pi_{wj}]$ reflects the elasticity of the customer's choice: the number $min\,\pi_{wj} - u^o$ is thus a measure of a "risk" that the customer will get into non-equilibrium situation. Clearly, a customer with a small interval will have grater difficulties to maintain the equilibrium than a customer with a wide interval.

5. Final remarks

It ends where we started. The paper investigated a situation of distributing commodities in the network with participants making "*to buy and sales*" decisions in a stream. One type of participants' produce and sale, others buy and sale, the third only buy for consumption. The price system was set up via some constants, which are nothing else than percentages to perform calculus of how the sales price must depend and exceed the purchasing prices to archive a satisfactory results for participants maximizing their profits. The situation becomes complex as soon as to buy and sale decisions incorporated transaction costs. Transaction costs, which originally have been called as overhead expenses, interact into the behavior of participants by transforming potentially profitable into loosing transactions. The paper investigated the situation as global depending on the transaction cost parameter varying the parameter from lower to high values until all transactions allegedly profitable became loosing and do not any more form a basis for an agreement between rational participants. The network structure, while the transaction cost parameter is increasing, changes like nested set of coalitions each of them on the higher level is capable to counteract higher transaction costs and still functioning in equilibrium. Condition for such a rational behavior was that all participants in the network must avoid loosing transactions. Beyond the goal of the coalition formation to hold the network in equilibrium, some elasticity intervals for transaction costs, where it still was realistic to buy and sale rationally, have been internally encoded into the scheme and calculated individually for all participants in the network.

6. Acknowledgment

This is a rewritten and expanded version of the paper appeared in “Automation and Remote Control” (“Automatika and Telemekhanika,” ISSN 005-1179) in 1980a, a Russian periodical of MAIK, Nauka/Interperiodica Press. An expanded introduction and conclusion sections reflect the connection of the paper with the New Institutional Economics and the theory of firm formation as it now became clear for the author. However, substantial amendments in terminology include also the main text of Mullat (1980b) version of the paper distributed by Plenum Publishing Corporation in English.

7. Appendix

Proof of the Lemma. Let S^o be a set-theoretically largest choice among all the joint choices rational with the threshold u^o, i.e., S^o is the largest choice H among all the choices such that $\pi_{wj}(H \bigcap R_{k+1},\dots,H \bigcap R_n) \geq u^o$.

Suppose that some customer p achieves a profit higher than Π_p by making the choice $A^p \subseteq R_p$, which is different from $S^o \bigcap R_p$; $\Pi'_p = \sum_{w\in A^p}\left(\pi_{wp}(\dots,A^p,\dots) - u_{wp}\right) > \Pi_p$, subject to $u^o \leq u_{wp} \leq min_{w\in A^p}\, \pi_{wp}$.

Clearly, the choice A^p is not a subset of S^o, since this would contradict the monotone property (8), so that $A^p \setminus S^o \neq \varnothing$. By the same monotone property, the customer making the choice $A^p \bigcup \left(S^o \bigcap R_p\right)$ will achieve a profit not less than Π'_p. On the other hand, all transactions in $A^p \setminus S^o$ are losing transactions for this customer, since S^o is the set-theoretically largest set of non-losing transactions for an overhead threshold $u^o > 0$. For the customer p making the choice $A^p \bigcup \left(S^o \bigcap R_p\right)$ the profit Π'_p does not decrease only if the total increase in utility due to the contribution π_{wp} of the transactions $w \in S^o \bigcap R_p$ exceeds the total negative utility due to the transactions in $A^p \setminus S^o$. Clearly, because of the constraint (9), the customer p has no such opportunity. This contradiction establishes the truth of the lemma. ■

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