A Puzzle about the Mass-Energy Composition of the Universe:
Narrative applying the Concept of FLRW ¹ Metric

J. E. Mullat
Independent researcher, Byvej 269, 2650 Hvidovre, Copenhagen, Denmark, mailto: mjoosep@gmail.com

Abstract. According to the Planck Mission Team, and based on standard cosmological model of known universe, the current composition of total mass–energy is comprised of 4.9% ordinary matter, 26.8% dark matter and 68.3% dark energy. By calibrating a speculative equation of matter composition, the equation confirms observable consequences of our theoretical insights as the equation solves for roots corresponding to 4.84% ordinary matter, 26.66% dark matter, and 68.50% dark energy what was considered a perfect match to the Planck mission statement. Regarding the past, the equation also confirmed some of those of Nasa Science Astrophysics statements.

Key words: Universe Composition, Dark Matter, Red Shift, Visible Matter, Dark Energy

1. Introduction

By an article published in Tallinn Technical University Proceedings [1], Estonia, there was presented a mathematical model considering maximization problem pertaining to functions defined on subsets of a set as the function argument. In 1979, the same mathematical problem was revised into a framework of stable or equilibrium subsets [2]. The framework was called the “The Monotone System—the MS.” The MS idea was applied to welfare economics [3]. MS also found applications in survey data and cluster analysis, game theory, matching problems, and among many other areas of research [4].

In trying to implement the MS idea to Universe Composition, we propose a Speculative Thesis that a new matter must apparently appear as soon as equilibrium has been established, since the old matter composition violates the MS stable set or equilibrium. Thus, based on this premise, the previous stable set violates itself and a new expansion of universe take place. However, this expansion achieves lower quasi-density compared to that of the previous state. The violation takes place due to monotone property of a gravitational potential function providing a theoretical fundament for the equation. In view of this thesis, the declining quasi-density of matter serves as an indicator of the universe expansion.

To implement the Monotone System idea, we are attempting to identify some stable subsets as 3-dimensional globes within the 4-dimensional geometry of curvature radius a, which can be expressed as:

\[ x_1^2 + x_2^2 + x_3^2 + x_4^2 = a^2. \]

In our view, this 4-dimensional globe is related to FLRW metric.

Finally, a few words about the method used. First of all, while solving the speculative equation, in order to calibrate the equation, the roots must point with great accuracy at the latest data of the mass-energy composition in the universe. Second, moving from large values of the quasi-density to their low values, the roots should confirm, or at least not be in stark contrast to the already known statements about the composition. Indeed, despite the not so deep knowledge of the subject as we would like to implement, our initiative appears to be successful. This allows us to believe that in addition to gravity and cosmology, since the method led to discovery of a number of new features in FLRW metric, the specialists will be able to apply the method to other fields of physics.

2. Theoretical backgrounds

Given the gravitational constant \( G = 6.67384^{-11} \cdot \text{m}^3 \cdot \text{kg}^{-1} \cdot \text{sek}^{-2} \), the curvature \( a > 0 \) and \( G \) are supposed to be constrained by \( G \cdot a^3 = 1 \). Irrespective of the interval scale adopted, i.e., (km)³ or (light year)³, etc., the curvature \( a \) and constant \( G \) must match—

¹ FLRW—Friedmann-Lemaître-Robertson-Walker
e.g., for $G = 6.67384^{-11}$, it follows that $a = 2465.32816; \text{ km}^3 \text{ scale } G = 6.67384^{-20}$ yields $a = 2465328.16025$, etc. The curvature $a$ might be fine-tuned to fit into the size of the universe. Considering the universe characterized by a positive curvature, it makes no difference what value $a > 0$ takes, as choices of $a$ and $G$ are transparent. However, when constraining the curvature $a$ the validity of $G \cdot a^3 = 1$ is crucial.

Let turn our attention to the energy level $u$ on the globe surface that forms geometry of 3-vectors $(x_1, x_2, x_3)$, i.e., to the level $u$ of potential energy at the distance $r$ from the centum of the globe of radius $r$. As already supposed in our speculative thesis above, there is a change in matter composition, which allegedly occurs on the 3-dimentional surface of the globe—more precisely, at a particular energy level $u$, defined by the expression $u = -G \cdot \frac{M(r)}{r^\lambda}$, where the $\lambda$ represents a fine-tuning or calibrating parameter. In accordance with the thesis, let the matter composition change occurs at energy level $u$ equal to $u = -\Lambda$. Thus, following $u = -\Lambda$, the change of matter composition occurs by violating the equation $-G \cdot \frac{M(r)}{r^\lambda} + \Lambda = 0$, where $M(r)$ does correspond to the mass of a globe of radius $r$. This equation represents the stable set equilibrium applied to the matter composition phenomena. Below, we will have to replace $M$ by $M = U(r) \cdot \mu$, where $U(r)$ is the volume of the globe allegedly expanding. Hereby, we refer to the parameter $\mu$ as a quasi-density of matter. Consequently, the composition equation might be rewritten in the form $-G \cdot U(r) \cdot \mu + \Lambda \cdot r^\lambda = 0$.

The surface rod $ds^2$ of 3-dimensional globe of radius $r$, $0 \leq r < a$, in accordance with (1), yields $ds^2 = \left(1 - \frac{r^2}{a^2}\right)^{-1}dr^2 + r^2(d\phi^2 + \sin^2 \phi \cdot d\theta^2)$. The transformation of the rod to $ds^2 = a^2\left(1 + \frac{\rho^2}{4}\right)^{-2}\left[d\rho^2 + \rho^2(d\phi^2 + \sin^2 \phi \cdot d\theta^2)\right]$ that is similar to Friedmann-Lemaître-Robertson-Walker—FLRW metric, is achieved by the replacement of radius $r$ to $r = a \cdot \rho \cdot \left(1 + \frac{\rho^2}{4}\right)^{-\frac{1}{2}}$, hereby the volume rod $ds^3 = a^3\left(1 + \frac{\rho^2}{4}\right)^{-5} \rho^4 \varphi \cdot \sin(\phi) \cdot d\theta \cdot d\phi$, $0 \leq \rho < \infty$, $0 \leq \phi \leq \pi$ and $0 \leq \theta \leq 2\pi$. Hence, $s(\rho) = \frac{2}{\pi} \int_0^{2\rho} \left(1 + \frac{\xi^2}{4}\right)^{-\frac{3}{2}} \xi^2 d\xi$ in 2$\rho$ reference system, i.e., $\rho \leftrightarrow 2\rho$ in $r$, provides a share $s(\rho) = \frac{2}{\pi} \left[\tan^{-1}(\rho) + \rho \frac{\rho^2 - 1}{(\rho^2 + 1)^2}\right]$, occupied by 3-dimensional globe space $U(\rho) = 2\pi^2 a^3 s(\rho)$ of radius $\rho$, in respect to the whole globe of $2\pi^2 a^3$. With regard to $U(\rho)$, the composition equation can be rewritten as:

$$-4\pi \cdot G \cdot a^3 \left[\tan^{-1}(\rho) + \rho \frac{\rho^2 - 1}{(\rho^2 + 1)^2}\right] \cdot \mu + \Lambda \cdot \rho^\lambda = 0,$$

(2)

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3. Rejection attempts

The Case A. Calibration. Parameters $\Lambda$, $\lambda$, and $\mu$ represent a triplet in (2), where $\Lambda$ is a mass-energy emerging level speculatively characterizes vacuum, $\lambda$ as a tuning or calibrating parameter for allegedly potential energy field, and $\mu$ denoting the quasi-density of the universe. By constraining $G \cdot a^3 = 1$, we have succeeded in calibrating the roots of the equation adopting $\Lambda = 0.91289$, $\lambda = 0.83641$ and $\mu = 0.12465$, as this parameter value set provides the best fit to the Planck team statement. Indeed, Equation (2) almost always resolves for two roots $\rho_0 < \rho_1$. However, the case with only one root $\rho_0 = \rho_1$ also exists, or the case of no roots at all. For $\Lambda = 0.91289$, $\lambda = 0.83641$ and $\mu = 0.12465$, the roots $\rho_0 = 0.67376$, $\rho_1 = 3.08377$, solve the equation. It can thus be verified that:

$$\text{vm\% } |s(\rho_0) \approx 26.66\%, \text{ de\% } |s(\rho_1) - s(\rho_0) \approx 68.50\%, \text{ dm\% } |s(\infty) - s(\rho_1) \approx 4.84\%.$$

These percentages, with regard to Plank mission statement, allow us to refer to $\rho_1$ as a visible matter start point, which terminates at $\infty$. Similarly, we can refer to $\rho_0$ as the dark energy starting point, while the dark energy terminates when it reaches $\rho_1$. We also refer to $r_0 - r_1$ as the dark energy width in $r$-reference system $r = 2a \cdot (1 + \rho^2)^{-1}$, see $\rho \leftarrow 2\rho$ substitution above, with $\rho_0$ and $\rho_1$ as its starting and ending points, respectively.

The Dark Energy width dynamics

The dark energy width 838.093 corresponds to $Vm$, $Dm$ and $De$ %-s, hereby the quasi density of our model is $1.43215$ times more dense in current state, when the dark matter accerelation related to visible matter is slowing down. The dark energy level equals $-0.91289$ where the matter creation allegedly occurs.
From the above, it can be inferred that, while the percentages provide a perfect match to the Planck Team statement, the roots $\rho_0$ and $\rho_1$ only fit the statement when $\mu = 0.12465$. Whatever the value $\mu = 0.12465$ of the quasi-density parameter proposes or is interpreted to imply, our supposition points at $\mu = 0.12465$ as an alleged current state of the universe. We also posit that this view cannot be discarded by statistical argumentation as false.

The Case B. Conjecture. The conclusion here is based on the premise that, in accord with our speculative thesis, the universe must stop changing its composition when the quasi-density reduces below the threshold $\mu < 0.08704$. In this case, the dark matter will collapse into or be in touch with the visible universe when $\mu \approx 0.08704$ because $\rho_0 \approx \rho_1$. Indeed, let us introduce a ratio reference scale ranging from critical quasi-density ratio $\mu \cdot e^{-1} \approx 1, e \approx 0.08704$. On this scale, the current composition of the universe points at the ratio $\approx 1.43215$. In contrast, for very high values $\mu \approx 100$ of quasi density the composition points at no visible $\text{vm} \% \approx 0.00000\%$ and a bit of dark matter $\text{dm} \% \approx 0.00193\%$. If the quasi-density decreases below the critical level $e \approx 0.08704$, the roots of the equa-
tion cease to exist and the alleged composition in our calculus when $\rho_0 \approx \rho_1$ suggests: $\text{vm\%} \approx 32.7\%$, and $\text{dm\%} \approx 67.3\%$. This last opportunity $\rho_0 \approx \rho_1$ for the equation to have a root is reached when $\mu \cdot \mathcal{E}^{-1} \rightarrow 1$; the energy width $\rho_1 - \rho_0 \rightarrow 0$. Therefore, the roots of the equilibrium in our speculative equation (2) do not contradict, but rather confirm the Nasa Science Astrophysics statement that the current composition of the universe is ca. 1.02 away at the point when the universe starts, as hypothesized, to expand forever.

The Case C. Conjecture. Note that the geometry (1) we examined above, as usually agreed in cosmology, does not contain the space-time coordinate. Therefore, the time coordinate is meaningless. Instead, we adopt the quasi-density parameter $\mu$, which is declining from very high $> > 13$ values—values $6 \cdot 10^4$ more than that of $\mathcal{E}$. Next, we attempt to move the quasi-density towards the critical value $\mathcal{E} \approx 0.08704$. Replacing the time by the quasi-density $\mu$ parameter is intuitive, due to the scale of densities, where declining values replicate the dynamics of universe expansion. Our calculus shows that, as the quasi-density $\mu$ declines towards the current composition of the universe, it accounts for the $\mu$ value pertaining to the current composition, which is only $1.43215$ times more dense than $\mathcal{E} \approx 0.08704$. 

![Critical Density of the Universe](image-url)

**Figure 3**

Critical Density of the Universe

- **C = 0.08704** Critical quasi-density
- $\xi = 1.33039$ $\rho_0 = \rho_1 = \xi$
- $-G \cdot \frac{U(\xi)}{\xi^\lambda} \cdot C = -0.91289$
- $\min(\Lambda) = -0.91289$
To our knowledge, or lack thereof, the Hubble’s low is a relationship between the velocity $v$ and the redshift of a galaxy at proper distant $\rho$ over time $t$. In other words, $v = H_0 \rho$, where $H_0$ is the Hubble’s constant, $H_0 \approx \frac{67.15 \text{ km/s Mpc}}{15.67}$. We reproduced, with the help of MathCAD spreadsheets on the quasi-density scale, the same effect in our calculus. However, our geometry allegedly claims that in the past, the visual $v''(\mu) > 0$ and the dark matter $d''(\mu) > 0$ accelerations were in swap order in relation to the current 1.43215 more crumbly composition. This swap leads to unexpected puzzle.

Indeed, the puzzle exhibits suggest that the dark and visible universe looks like expanding along a 3-dimentional Mobius strip but from its opposite sides: at the radius $\rho = 0$ and $\rho = \infty$. If an observer in the past was inside the dark part of the strip, the dark matter would be allegedly shifting away from the visible matter. Yet, at some "turn-around or speeding up point," the visible matter gained speed starting to shift towards the dark matter. Thus, the visible matter started to come increasingly closer to the dark matter because the expansion acceleration of the dark part might have started to decline in relation to that of the visible matter. While both—the visible and the dark matter—are still expanding with acceleration, the visible matter, hereby, will collapse into or be in touch with the dark matter at some future point, as discussed above. After the collapse, both the dark and the visible matter must complete their expanding process. However, the conjecture claims that the dark matter at the collapse point changed to negative acceleration, i.e., merged back.

Our calculus suggests that the turnaround point allegedly occurred in the past when quasi-density on the ratio scale was approximately three times of the critical density $\epsilon \approx 0.08704$. Currently, our geometric equation resolution allegedly claims that $\mu = 1.2465$ and the universe had already passed the turnaround point. This claim can neither be rejected nor confirmed by observations. Most importantly, beyond all these aspects, this speculative claim does not contradict, but rather supports, the NASA SCIENCE ASTROPHYSICS statement: “Then came 1998 and the Hubble Space Telescope (HST) observations of very distant supernovae that showed that, a long time ago, the Universe was actually expanding more slowly than it is today.”
Dark Matter Acceleration Dynamic

\[ \frac{d^2}{d\theta^2} \cdot dm(\theta) = \theta \frac{\mu}{C} \]

Quasi-Density / Critical Density

- Current state of the universe \( r = 2284.85833 \)
- Critical Quasi-Density \( C = 0.08704 \)

The State of the Universe in the Past

Figure 4

Visual Matter Acceleration Dynamic

\[ \frac{d^2}{d\theta^2} \cdot vm(\theta) = \theta \frac{\mu}{C} \]

Quasi-Density / Critical Density

- Current state of the universe \( r_1 = 1446.76867 \)
- Critical Quasi-Density \( C = 0.08704 \)

The State of the Universe in the Past

Figure 5
The Case D. Conjecture. Here we are concerned with the relationship between radial coordinate $r$ and radial coordinate $\rho$:

$$r = \frac{2 \cdot a \cdot \rho}{1 + \rho^2}, \quad 0 \leq r < a, \quad 0 \leq \rho < \infty.$$ 

The relationship $\rho \rightarrow r$ represents a mapping between $[0, a)$ and $[0, \infty)$. Notice that, in accordance with this relationship $\rho \rightarrow r$, we observe that $\infty \rightarrow 0$, $0 \rightarrow 0$ and $1 \rightarrow a$. One can imagine this geometry by looking at 3-dimentional Mobius strip—actually $\rho = 0$ and $\rho = \infty$ point at the same location for an external observer but at allegedly opposite sides of the strip. Hence, the relationship $\rho \rightarrow r$ does not represent a one-to-one mapping. Given two roots $\rho_0, \rho_1$, which were calculated using the quasi density $\mu = 0.12465$, see above, the relationship looks like as: $\rho_0 \rightarrow \text{De}$, $\rho_1 \rightarrow \text{Vm}$. One can check that, due to absence of one-to-one mapping between coordinates $r$ and $\rho$, the dark energy starting at $\rho_0$ corresponds to the dark matter ending at $\rho_1$. Another part of the dark matter globe co-exists with higher values in radial $r$ reference system, which accounts for the visible matter of radius $\rho_1$. However, there is no common $r$ reference system for dark energy and visible matter.

We found the Nasa Science Astrophysics statement. “One explanation for dark energy is that it is a property of space. Albert Einstein was the first person to realize that empty space is not nothing. Space has amazing properties, many of which are just beginning to be understood. The first property that Einstein discovered is that it is possible for more space to come into existence. Then one version of Einstein’s gravity theory, the version that contains a cosmological constant, makes a second prediction: "empty space'' can possess its own energy. Because this energy is a property of space itself, it would not be diluted as space expands. As more space comes into existence, more of this energy-of-space would appear. As a result, this form of energy would cause the Universe to expand faster and faster. Unfortunately, no one understands why the cosmological constant should even be there, much less why it would have exactly the right value to cause the observed acceleration of the Universe.” Once again, we cannot reject that our puzzle exhibits contradict the observable consequences known at the moment.

4. Monotone System Theory on non-Euclidean geometry

In order to surmount with the problem of Big Bang singularity, we can assume that matter creation occurs from an unknown dark energy field. While a “bit of matter” suddenly emerges it will create an additional pressure on previously frozen energy thereby coursing an additional freezing effect. It is not wrong to imaging that there is some process like “avalanche” rolling down the hill and gaining weight due to “potential energy of dark energy field.” However, the avalanche process of the matter creation has to stay in a stable condition at some point. Note that the avalanche snowball effect creates the time as well. Mathematical representation of what has been said confirms the hypothesis that the singularity will not take place if the density of frozen matter is large enough. It is clear, after all, that a friable ball cannot roll down a slope, i.e., not lounging on its way down. Let see what mathematics can suggest in this direction.
Given by 

\[ ds^2 = a^2 \left( 1 + \frac{\rho^2}{4} \right)^2 \left[ d\rho^2 + \rho^2 (d\phi^2 + \sin(\phi)^2 d\theta^2) \right], \]

where \( 0 \leq \rho < \infty, \) 

\[ 0 \leq \phi \leq \pi, \ 0 \leq \theta \leq 2\pi, \ a > 0 \]
is the radius of positive curvature, we consider the continuum—a coordinate system \( W = \{ \rho \in [0, \infty), \phi \in [0, \pi], \theta \in [0, 2\pi] \}, \) represented by at most countable set of operations of union, intersection, and difference of segments/subsets \( \{ H \} \) of \( W, \ H \subseteq W. \) The class \( \{ H \} \) of all such subsets is denoted by \( \mathcal{B}, \) and each representative subset by \( H \in \mathcal{B} \) (which we call a \( \mathcal{B} \)-set) is distinguished from the like sets by length \( \mu \) (by measure zero). The measure \( \mu \) is supposed to be an additive function \( \mu(H), \) i.e., \( \mu(A \cup B) + \mu(A \cap B) = \mu(A) + \mu(B). \) A set \( L \) is congruent with \( G, \) \( G \equiv L, \) if the measure of the symmetric difference—\( A \Delta B = (A \cup B) \setminus (A \cap B)—\)is equal to zero: \( \mu(G \Delta L) = 0; \) a set \( G \) is contained in \( L \) (\( L \ni G \)) with respect to measure \( \mu \) if \( \mu(G \setminus L) = 0. \) A measure on the real axis, being an additive function of sets (the mass), is determined by taking to the limit the mass of the sets in the set of unions, intersections, and differences of segments forming the \( \mathcal{B} \)-set. Then, the set-theoretic operations over \( \mathcal{B} \)-sets will be understood to mean up to measure zero. By convention, all \( \mathcal{B} \)-sets of measure zero are indistinguishable.

We associate with every \( \mathcal{B} \)-set \( H \) with positive measure \( \mu(H) > 0 \) a negative function \( \pi(\alpha; H), \ \alpha \in H, \) which is Borel measurable (or simply measurable) and whose domain of definition is on the real axis. A function \( \pi(\alpha; H) \) is Borel measurable if for any negative threshold \( u \) the set of all those elements \( \alpha \in H \) for which \( \pi(\alpha; H) \leq u—\)the set \( \{ \alpha \in H | \pi(\alpha; H) \leq u \}—\)is a measurable \( \mathcal{B} \)-set. We say that a family of measurable functions \( \{ \pi(\alpha; H) \} \) is monotonic if it obeys the following condition: for any pair of sets \( L, G \) such that \( L \ni G \) the inequality 

\[ \pi(\alpha; L) \geq \pi(\alpha; G) \]

holds for any \( \alpha \in G \).

We consider in Monotone System over \( \mathcal{B} \)-set the following mapping of sets \( X \in \mathcal{B}: \)

\[ V_u(X) = \{ \alpha \in X | \pi(\alpha; X) \leq u \}. \]

Consider a stable set or equilibrium equation \( X = V_u(X). \)

**Observation 1.** Given a pair of stable sets \( X_1 \) and \( X_2, \) there always exists a stable set \( X \ni X_1 \cup X_2. \)

**Definition.** A stable set \( X^* \) of a positive measure \( \mu \) at the minimum level of the parameter \( u: X^* = \arg \min_{x \ni V_u(X)} u \) is called kernel.

**Corollary.** The set of all kernels arrange an upper semilattice of sets, i.e., for any pair of kernels \( X_1^* \) and \( X_2^* \) there exists a kernel \( X^* \ni X_1^* \cup X_2^*. \)
5. Concluding remarks

We presented a speculative equilibrium equation that describes the matter composition while emerging from dark energy, and still continuing to emerge. In our analyses, we did not claim in any way that the proposed scheme is applicable to the Universe mass-energy composition of the standard $\Lambda$CDM model. Finding the equation roots might have some utility, since they yield an almost perfect match to the Planck mission statement. Indeed, all our statements presented here are in correspondence with the latest measurements of the data composition between the percentages of visible and dark matter in proportion to the dark energy. This correspondence is achieved due to the calibration that the radius of curvature of the space and the gravitational constant are related by $G \cdot a^3 = 1$. This ensures the independence of outcomes of the visible and dark matter fractions in proportion to the dark energy, in case the gravitational constant $G$ interval scale of measurement is changed (i.e., it guarantees the correct output irrespective of whether $G$ is measured in meters, kilometers, or any other scale). Our speculative equation requires also an additional calibration of the following parameters: $\Lambda$ -parameter of alleged potential energy, and the so-called $\Lambda$ -parameter of speculative mass-energy emerging level. This allows the optimal values to be determined, with respect to achieving the best tuning effect of the Planck mission statement. The next important assumption was the density parameter $\mu$ of the emerging matter, to which we referred as quasi-density. While acknowledging that the explanation of the density requires more convincing arguments, we proceed like the quasi-density is in lines with normal density of matter. However, the concept of quasi-density allows interpreting, as well as predicting, the dynamics of expansion and acceleration of the universe. It was possible to make assertions that essentially coincide with the NASA statement that, in the past, the universe expanded more slowly than it does presently. Moreover, as our geometry implies, only a tiny globe of dark matter solves the equation at high values of the quasi-density. Thus, at high quasi-density, the geometry comprises almost totally of dark energy since the visible matter radius suggests almost a zero solution. On the low side of the scale, approaching the critical value, the dark matter, in contrast to the visible matter, is going allegedly to scrunched.

Summary. First, the percentages of the visible and dark matter in their proportions to dark energy yielded almost perfect match with the latest Planck mission data. Second, it was claimed that the universe is expanding, with decreasing quasi-density, but with increasing acceleration. This has been established from the equilibrium equation (2), the outcomes of which confirm that the universe was also expanding more slowly in the past. Third, there is a critical value of the quasi-density at which the dark energy will be exhausted. When this occurs, the universe will allegedly collapse into the critical composition. However, it will do so later than predicted by the standard cosmology model. In critical composition the dark matter will rather scrunched than expand in contrast to visible matter, which will continue to expand. Finally, there is a likelihood that the dark matter coexists with the visual matter, since there is a correspondence—like on Mobius strip—for the latter located at the large distances of Friedmann-Lemaître-Robertson-Walker metric, i.e., for the observers located in the visual space at the radius $\rho$, as opposed to the radius $r$ reference system in the geometry of (1).

References